Leadership in Collective Action

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Abstract

We merge the current approach to collective action failure based on free-riding (Olson, 1965) with the previous approach focusing on the distortions created by the need of leaders (Michels, 1911 and Max Weber, 1918). Group goals and incentives are set by leaders in view to maximize the probability of success —rather than the group expected payoff. In spite of not being group optimal, success maximizing leaders introduce incentives. Because there are incentives, leaders also distort the goals of the group away from the ideal provision of public good and devote excessive resources to private compensations. The distortion introduced on goals by leaders increase with group size. This creates an additional source of inefficiency in collective action.

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1 Introduction

How a group of individuals manage to pursue some common end has always been a central theme in the social sciences: the collective action problem. The influential work of Olson (1965) emphasized the now widely studied free-rider problem in collective action: individuals contribute less than would be optimal for the group because the costs of participating in the group's activity are privately incurred but the individual's benefit depends on the collective activity of the entire group she is part of.[1] Research has essentially concentrated on ways to mitigate the effects of free-riding,[2] on the importance of this problem with respect to group size[3] and on the endogenous formation of competing groups.[4]

The idea of individual free-riding has been so powerful and inspiring that it has overshadowed the previous analyses of the collective action problem. Instead of focusing on the money or effort contributions by the individual group members, Max Weber (1918, 1925) and, most notably, Michels (1911) centered their work on the inability of groups to coordinate to set a platform. This inability to coordinate was solved by the action of group leaders. In Weber's (1925) words 'Politics is always made by a small number of persons' (1968 translation, vol. 3, p.1421).[5] The main point by Michels —

[1]This argument can be traced back to Pareto (1927). See also Hardin (1968) who coined the term "tragedy of the commons". The literature on collective action failure has been surveyed by Sandler (1992).

[2]Monitoring and sanctions may alleviate this problem. Experimental evidence is examined in Andreoni et al. (2003), Fehr and Gächter (2000) and Masclet et al. (2003) and field evidence in irrigation systems in India in Bardhan (2000).

[3]Olson (1965) formulated the so-called "group size paradox" that smaller groups should be more effective in resolving the free-rider problem. Esteban and Ray (2001) shows that the opposite will hold if contribution costs are sufficiently convex.

[4]Several papers have studied this problem: see Anesi (2007), Bardhan and Singh (2004), Esteban and Ray (2008), and references therein. Anesi (2007) adds a lobbying-formation game a la Mitra (1999) (where forming a lobby involves a fixed cost F_i for group i) to the model of Esteban and Ray (2001). Anesi shows that the typical free-riding problem when lobby members make their effort decisions (which he calls moral hazard in teams) may raise large groups' equilibrium lobby size and also the total contribution to lobbying of large groups with low organizational costs. The possibility to free-ride on the effort levels of one's fellow lobby members lowers the cost of being a member of the lobby and thereby increases participation in lobbying activities. In other words, moral hazard in teams decreases individual contributions to lobbying but raises the number of contributions.

[5]Schumpeter (1942) also viewed the utilitarian idea of welfare maximizing political parties —let alone politicians— as utterly unrealistic. Following Weber and Michels,

his famous "iron law of oligarchy"— was that group effectiveness demanded leaders; leaders would spontaneously create a paid bureaucracy around them with the corresponding incentives;[6] and they would end up by distorting the group's genuine ends in an attempt to secure their continuation in power.[7] In other words, professionalized leaders surrounded by a paid bureaucracy end up by setting objectives that are a compromise between the original goals and their own private interests.[8] This is what Michels calls the "oligarchy" that gets imposed in the name of efficiency even in democracy seeking organizations. For this line of thought, the main source of inefficiencies in collective action derived from the unavoidable need for leaders.[9] Again in Weber's (1918) words "This is simply the price paid for the guidance of leaders" (2000 translation, p. 113).

These two lines of thought are clearly complementary, not substitutes. Free-riding occurs because individual members fail to coordinate in an action and so they should also be unable to coordinate to fix common ends. Conversely, the need for leadership reveals the inability of a group to self-organize and consequently to collectively commit to the group optimal action. In this paper, we bring together the two aspects of collective action failure and study their interaction. In a unified model we let individuals free-ride and at the same time the group platform be set by leaders.

The introduction of group leaders gives a new twist to the collective action problem. Opportunistic leaders only seek to maximize their probability of success rather than the group's expected payoff. But to serve their goal they need the group members to deliver high contributions and hence are interested in alleviating as much as possible the free riding problem. Col-

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he also viewed collective action as organized by leaders. It is the competition among leaders/political-entrepreneurs what could safeguard groups from a purely selfish exploitation by the leaders.

[6]Oliver (1980) sees as a key functions of the leaders "to dispense selective incentives (...) to members of the group based upon their cooperation with or defection from contributing to the collective good."

[7]See Scaff (1981) on the influence of Weber on Michels on this point. Ansell (2007) provides an excellent summary of Michels "iron law".

[8]This process appears to be common to most organizations. Selznic (1984) attests this pattern in his study of the Tennessee Valley Authority. On the general point of the displacement away from the original goals see Cazessus and Steketee (2006).

[9]On leadership in collective action, see Frolich et al (1971), Calvert (1992) and Colomer (1995). Cai (2002) empirically examines the characteristics of the community that are conducive to group decision or to external leadership.

lective action appears now as resulting from two opposing forces. On the one hand, individuals still benefit from the collective effort by the group and do free ride. But on the other hand, seeking their own interest, leaders will adapt the goals and set incentives so as to induce individual members to contribute to the common cause. The balance between the two forces will permit to establish the costs or benefits of leadership and hence the sign of its contribution to the collective action problem.

We formalize the collective action problem as follows. There are a number of groups competing for the control of a given budget. The win probability is determined by the relative resources contributed by the individual members of each group. These contributions are private and hence individuals free-ride on the effort of their fellow group members. The leader of each group determines the group platform consisting of the split of the budget between the public and the private good[10] and the allocation of the private good across membership. While the group efficient platform would be the one yielding the highest expected payoff, group leaders, driven by their thirst for success, choose instead the platform that maximizes the probability of controlling the budget. Therefore, even though leaders' interests are not perfectly aligned with the "rank-and-file", they nevertheless seek to maximize the win probability of the group.[11] This is the displacement of goals unveiled by Michels and the subsequent literature. The eventual creation of a paid bureaucracy is captured in our model by the choice of introducing incentives based on individual contributions as opposed to a uniform distribution independent of effort.[12] In sum, we have a two-stage game in which first leaders choose the group platform —including the use or not of incentives— and then group members privately decide how much effort to contribute. We study the Nash equilibrium of this game. In order to evaluate the losses imputable to leadership we contrast the equilibrium outcome with the one that would result if groups were able to coordinate on their goals (but individual contributions

[10] Cornes and Sandlers (1984, 1994) and Esteban and Ray (2001) also develop conflict models with mix public/private payoffs. In both cases, the mix is exogenous.

[11] Knoke (1990, p.15) pointed out that leaders have to respect the broad interest of their constituency because participation in collective action tends to be voluntary and leaders must continuously mobilize constituency support from their members.

[12] Olson (1965) already mentioned the introduction of incentives as the way to solve the free-riding problem. Lee (1995) and Ueda (2002) examine rent-seeking models in which groups can choose their own incentive scheme. Bandiera et al. (2005) obtain experimental evidence on the effects of using piece-rate in collective action.

continued to be privately decided).

Our results are the following. The game has a unique equilibrium in which all leaders choose to use incentives and a degree of privateness in the budget allocation that exceeds the one that would be group optimal. This degree of privateness is increasing in group size, thus adding an extra source of inefficiency for large groups. Aggregate effort/waste is maximal. The divergence in the goals of the groups and of their leaders materializes in biased platforms only because of the introduction of incentives. The introduction of incentives is not optimal to the group: the group's expected payoff is higher under an egalitarian distribution of the private good. If leaders could not distribute the private good on the basis of performance, their choice of platform would coincide with the group optimum. However, leaders can increase their win probability by introducing incentives and hence will do so. It is then when leaders also choose a level of privateness that exceeds the group optimum. Since probabilities have to add up to one, the introduction of incentives will produce winners and losers even among success seeking leaders. Comparing the equilibria with and without incentives we find that large groups will see their win probability increased (and small groups reduced). Hence, if leaders could foresee the full general equilibrium consequences from the introduction of incentives, the leaders of the large groups would be the ones who would precipitate the shift from egalitarianism to incentives.[13]

Concerning the old discussion on whether larger groups are in a disadvantage, we obtain that through the action of leaders the win probability is increasing in group size. However, this is at the cost of devoting a larger part of the budget to incentives. High win probabilities are "purchased" with lower supply of public goods and a lower individual expected payoff.

The remainder of the paper is organized as follows. In Section 2 the model is presented. Section 3 solves for the equilibrium of the private contributions in the second stage of the game. Section 4 is devoted to the choice of platform in the first stage of the game. We start by solving the benchmark case of group decision and then derive the platform and distribution rule that would be chosen by an opportunistic leader. The efficiency properties of the platform are discussed and it is shown how the choice of the platform

[13]This result implies that the formation of an "oligarchy" in the German Social Democratic Party was not only caused by the complexity of the organization (as argued by Michels and Weber), but because of the higher benefits to leaders in large groups. From our argument it also follows that smaller groups, like the communist party, had no other best reply but to create a paid core organization as well.

varies with group size and its effect on win probabilities is derived. Section 5 puts together the different results obtained and identifies the different losses generated by leadership. Section 6 concludes by relating our results with existing evidence. Proofs are relegated to a technical appendix.

2 The Model

Suppose that there are G different types of public goods and the same number of types of preferences, accordingly with the type of public good individuals prefer.[14] Individuals are assumed to derive utility from their own type of public good only. Let n be the total population and $n_1, n_2, ..., n_G$ be the population of the G types of preferences. Without loss of generality, we assume that $n_i \leq n_{i+1}$.

We assume that people with the same preferences form a group. Each group is organized as a lobby (or political party), competing with the opposing groups in view of controlling the allocation of the public budget b. Group competition is modelled as a two-stage game. In the first stage the group platform is fixed and in the second stage individuals privately decide how much to contribute.

The group platform has two ingredients: (i) the share λ of the budget b to be allocated to the production of the group-specific public good, and (ii) the specification of how the private good produced with the remaining budget will be allocated among the group members.

We will study two different ways in which the private good is transferred to the group members: an egalitarian split and transfers intended to provide incentives for collective action. In the latter case the platform specifies that each individual receives a proportion of the group money equal to their share of total group effort. Following Nitzan (1991) we also call the uniform split the *egalitarian rule* and the split linked to incentives the *relative effort rule*.

Platforms are set by group leaders who extract utility from victory and the control of the budget. We normalize their payoff to 1. Hence, the expected payoff to a leader is the win probability p. Put differently, leaders only care about the probability of success and therefore choose the platform in view of maximizing the win probability of the group they lead irrespective of the private cost of the contributions to the group members.

[14] Our model extends the models of Esteban and Ray (2001) and of Banerjee et al. (2008).

In the second stage, in view of the group platform and of the contributions by the others, individuals decide how much to contribute to the collective cause. Individual contributions determine the win probabilities of each group. Finally the winning group is chosen by nature.

Individuals contribute effort r_{ik} in support of the platform of their group where i stands for the group and k for the individual. We choose these units of effort so that effort is *added* across group members to yield group effort R_i of group i. As in Esteban and Ray (2001) we model the utility cost of effort $c(r)$ as an increasing smooth, convex function with $c'(0) = 0$ and $c'(\infty) = \infty$, and with an elasticity $\eta(r)$

$$\eta(r) = \frac{rc''(r)}{c'(r)} \geq 1.^{15} \tag{1}$$

We make the standard assumption that the probability that group i wins, p_i, equals the effort level of group i, R_i, relative to the aggregate amount of effort R exerted by *all* groups. Therefore, we have that

$$p_i = \frac{R_i}{R} = \frac{\Sigma_k r_{ik}}{R}.^{16} \tag{2}$$

Individual preferences are additively separable in the concave valuation of the public good, $v(.)$, and the linear valuation of the private good. On $v(.)$ we assume it be strictly concave with the Inada limit conditions: $\lim_{z\to 0} v'(z) = \infty$ and $\lim_{z\to\infty} v'(z) = 0$. Furthermore, we assume that $v'(b) \leq \frac{1}{n_G}$. This assumption simply establishes the size of prize b relative to the (largest) group size.

As already mentioned, we will study two different rules how to distribute the private good. The first rule, the egalitarian rule, consists of an egalitarian division among all group members. In this case, individual decisions cannot modify the size of the transfer she receives. The second rule, the relative effort rule, establishes incentives to reward the effort contributed; each group member receives a proportion of the group money equal to their share of

[15] This elasticity plays a crucial role for the main results of Esteban and Ray (2001). In particular in their model $\eta(r) \geq 1$ is a necessary and sufficient condition for the unconditional reversal of Olson's results, that is, that the win probabilities be strictly increasing in group size irrespective of the group platform.

[16] Notice that the win probability is not defined when $R_j = 0, \forall j$. We shall simply assume that in this case $p_i = \frac{n_i}{n}$.

total group effort. Hence, the transfer received $\frac{(1-\lambda_i)br_{ik}}{R_i}$ does depend on individual's decisions r_{ik}.

In the case of egalitarian transfers the expected utility u_{ik} of member k of group i is given by

$$u_{ik} = p_i \left(v\left(\lambda_i b\right) + \frac{\left(1-\lambda_i\right) b}{n_i} \right) - c(r_{ik}) \tag{3}$$

and with incentives it is

$$u_{ik} = p_i \left(v\left(\lambda_i b\right) + \frac{\left(1-\lambda_i\right) br_{ik}}{R_i} \right) - c(r_{ik}). \tag{4}$$

For convenience we denote by $\omega_{ik}(\lambda_i, n_i, r_{ik})$ the payoff of member k of group i in case of victory. Therefore, either under egalitarian transfers

$$\omega_{ik}(\lambda_i, n_i, r_{ik}) = \omega_i(\lambda_i, n_i) = v\left(\lambda_i b\right) + \frac{\left(1-\lambda_i\right) b}{n_i}, \tag{5}$$

or under incentives

$$\omega_{ik}(\lambda_i, n_i, r_{ik}) = v\left(\lambda_i b\right) + \frac{\left(1-\lambda_i\right) br_{ik}}{R_i}. \tag{6}$$

In general, we will write

$$u_{ik} = p_i \omega_{ik}(\lambda_i, n_i, r_{ik}) - c(r_{ik}). \tag{7}$$

This game has two stages. First, groups or leaders choose the platform and then individuals privately decide how much to contribute. We solve the game backwards. In the next section we characterize the Nash equilibrium of the contributions game for given platforms that is played in the second stage of the game. In section 4 we shall deal with the first stage of the game.

3 Equilibrium Contributions in the Second Stage of the Game

In the second stage, given the group platforms, individuals choose their effort. We assume that each individual takes its best course of action, given the behavior of the rest of the population —both, fellow group members and the rest— and hence free-rides on the effort contributed by the other group members. Therefore, in view of (2) the effect of an increase in effort r_{ik} on the win probability will be strictly increasing and strictly concave. Indeed,

$$\frac{\partial p_i}{\partial r_{ik}} = \frac{R - \sum_k r_{ik}}{R^2} = \frac{1}{R}(1 - p_i) > 0.$$

and

$$\frac{\partial^2 p_i}{\partial r_{ik}^2} = -\frac{1}{R^2}(1 - p_i) - \frac{1}{R}\frac{\partial p_i}{\partial r_{ik}} = -\frac{2}{R^2}(1 - p_i) < 0.$$

Differentiating u_{ik} with respect to effort r_{ik} we obtain

$$\frac{\partial u_{ik}}{\partial r_{ik}} = \frac{\partial p_i}{\partial r_{ik}}\omega_{ik}(\lambda_i, n_i, r_{ik}) + p_i\frac{\partial \omega_{ik}(\lambda_i, n_i, r_{ik})}{\partial r_{ik}} - c'(r_{ik}). \tag{8}$$

The effect of r_{ik} on ω_{ik} depends on whether the sharing rule is egalitarian or the relative effort rule. In the first case the change in effort has obviously no effect on ω_{ik}. For the relative effort case ω_{ik} is strictly increasing and strictly concave in r_{ik}. Differentiating we have

$$\frac{\partial \omega_{ik}}{\partial r_{ik}} = (1 - \lambda_i)b\frac{R_i - r_{ik}}{R_i^2} > 0,$$

and

$$\frac{\partial^2 \omega_{ik}}{\partial r_{ik}^2} = -\frac{2}{R_i}\frac{\partial \omega_{ik}}{\partial r_{ik}} < 0.$$

We first show that the first order condition

$$\frac{\partial u_{ik}}{\partial r_{ik}} = 0 \tag{9}$$

implies a within-group symmetric equilibrium.

Lemma 1 *For each group i there is a unique r_i that satisfies (9) for each individual k.*

Proof See appendix.
Lemma 2 states that (9) indeed characterizes a maximum.

Lemma 2 *In each case —egalitarian and incentives— the unique r_i maximizing u_i is implicitly given by*

$$\frac{1-p_i}{R}\left(v\lambda_i b + \frac{(1-\lambda_i)b}{n_i}\right) - c'(r_i) = 0 \textit{ (egalitarian)} \tag{10}$$

$$\frac{1-p_i}{R}\left(v\lambda_i g + \frac{(1-\lambda_i)br_i}{R_i}\right) + p_i\frac{(1-\lambda_i)b(R_i-r_i)}{R_i^2} - c'(r_i) = 0 \textit{ (incentives)} \tag{11}$$

Proof See appendix.

Notice, that for given R determining r_i is equivalent to determining p_i. For future use, we rewrite the FOC that characterize the best response r_i under both sharing rules as:

$$\frac{1}{R}(1-p_i)\omega_i(\lambda_i, n_i) - c'(\frac{p_i R}{n_i}) = 0 \text{ (egalitarian)} \tag{12}$$

$$\frac{1}{R}(1-p_i)\left(v(\lambda_i b) + \frac{(1-\lambda_i)b}{n_i}\right) + \frac{(1-\lambda_i)b(n_i-1)}{n_i R} - c'(\frac{p_i R}{n_i}) = 0 \text{ (incentives)}, \tag{13}$$

where we have used the fact that

$$r_i = \frac{p_i R}{n_i}. \tag{14}$$

Expressions (12) and (13) implicitly define the win probability p_i as a function of the exogenous parameters (λ_i, n_i, b) and of the *endogenous* value of R. We shall write

$$p_i = \psi(R, \lambda_i, n_i, b) \tag{15}$$

for the two cases with egalitarianism and incentives. The following Lemma will be instrumental in proving existence and uniqueness of a Nash equilibrium of the contribution game.

Lemma 3 *$p_i = \psi(R, \lambda_i, n_i, b)$ is a continuous strictly decreasing function of R for all i.*

Proof See appendix.

Definition 1 *A Nash equilibrium of the second stage of the game is a vector* $\mathbf{p}^*$ *and a value* R^* *such that*

$$\sum_i \psi(R^*, \lambda_i, n_i, b) = 1 \tag{16}$$

and for all i

$$p_i^* = \psi(R^*, \lambda_i, n_i, b). \tag{17}$$

Notice that from the equilibrium p^*, R^* we can immediately obtain r_i^* for all $i = 1, ..., G$ using equation (14).

We can now establish the existence and uniqueness of equilibrium.

Proposition 1 *For every set of parameters* (λ, n, b) *there exists an equilibrium of the second stage of the game and it is unique.*

Proof See appendix.

It follows that we can write

$$R^* = \rho(\lambda, n, b). \tag{18}$$

Therefore, the equilibrium win probabilities are

$$p_i^* = \psi(\rho(\lambda, n, b), \lambda_i, n_i, b). \tag{19}$$

We are now set for the analysis of the choice of platform in the first stage of the game.

4 Choice of Platform in the First Stage

Leaders choose the platform that maximizes their probability of success, i.e. the win probability of their group. This objective does not coincide with the maximization of the expected payoff to the representative group member. In order to evaluate the potential bias introduced by entrepreneurial leaders we shall take the utility maximizing platform as a benchmark case. We shall contrast this *group optimal* platform λ_i^{ah} with the choice of platform made by an entrepreneurial leader, λ_i^{oh}, where $h = e, s$ stands for the egalitarian and the relative effort sharing rules.

When deciding on the platform, both leaders and groups that manage to self organize take into account that *all* group members will change their behavior in response to changes in λ_i. The best reply of each individual group member is known and this information is used when deciding on the desired platform. Notice that this best reply depends on the effort contributed by the members of the other groups —through the win probability— but not directly on the specific platform that they might have adopted. Therefore, an equilibrium will require that there exists a vector of probabilities such that all the associated platforms —and the individual contributions— be best responses to each other.

4.1 Preliminaries

An entrepreneurial leader will choose λ_i^{oh} so as to maximize p_i. The corresponding first order condition for a maximum implies that λ_i^{oh} solves

$$\frac{dp_i}{d\lambda_i} = 0.$$

The group optimal platform instead seeks to maximize the expected utility of group members. The first order condition implies that λ_i^{ah} solves

$$\frac{du_i}{d\lambda_i} = \frac{dp_i}{d\lambda_i}\omega_i + p_i\frac{d\omega_i}{d\lambda_i} - \frac{dc}{d\lambda_i}. \tag{20}$$

Developing the differentiation and reorganizing terms equation (20) can be rewritten as

$$\frac{du_i}{d\lambda_i} = \left(\omega_i - \frac{R}{n_i}c'(p_i\frac{R}{n_i})\right)\frac{dp_i}{d\lambda_i} + p_i\frac{d\omega_i}{d\lambda_i} - \frac{p_i}{n_i}c'\left(p_i\frac{R}{n_i}\right)\frac{dR}{d\lambda_i} = 0. \tag{21}$$

For leadership and our benchmark case we will need to compute $\frac{dp_i}{d\lambda_i}$. Differentiating (19) we obtain,

$$\frac{dp_i}{d\lambda_i} = \frac{\partial\psi(\lambda_i)}{\partial R}\frac{dR}{d\lambda_i} + \frac{\partial\psi(\lambda_i)}{\partial\lambda_i}.$$

We start by computing $\frac{dR}{d\lambda_i}$. Differentiating (16) with respect to R and λ_i we get

$$\sum_j \frac{\partial\psi(\lambda_j)}{\partial R}dR + \frac{\partial\psi(\lambda_i)}{\partial\lambda_i}d\lambda_i = 0.$$

That is

$$\frac{dR}{d\lambda_i} = -\frac{\partial\psi(\lambda_i)}{\partial\lambda_i}\frac{1}{\sum_j \frac{\partial\psi(\lambda_j)}{\partial R}} \tag{22}$$

Reorganizing and substituting above, we obtain

$$\frac{dp_i}{d\lambda_i} = \frac{\partial\psi(\lambda_i)}{\partial\lambda_i}\left(1 - \frac{\frac{\partial\psi(\lambda_i)}{\partial R}}{\sum_j \frac{\partial\psi(\lambda_j)}{\partial R}}\right). \tag{23}$$

By Lemma 1 we know that p_i is a strictly decreasing function of R. It follows that the fraction within the braces is positive and strictly less than 1. So, the braces are always positive and less than 1. Hence,

Remark 1 *The sign of* $\frac{dp_i}{d\lambda_i}$ *is equal to the sign of* $\frac{\partial\psi(\lambda_i)}{\partial\lambda_i}$.

Furthermore, in view of (22) we also have the following useful result:

Remark 2 *The sign of* $\frac{\partial R}{\partial\lambda_i}$ *is equal to the sign of* $\frac{\partial\psi(\lambda_i)}{\partial\lambda_i}$.

Note that in the derivation of (22) and (23) we have not made any assumption on whether the distribution of the private benefits was egalitarian or based on incentives. Hence, Remarks 1 and 2 are valid for both cases. These two remarks will be essential in characterizing the optimal platforms.

4.2 The Group Optimal Benchmark

We start with the benchmark case where the platform is chosen to maximize the well-being of the representative group member. We will first derive the group optimal platform under an egalitarian sharing rule λ_i^{ae}. Then we will show that the egalitarian sharing rule is part of the overall preferred group optimal platform. In other words we will prove that groups that are able to self-organize will not introduce the relative effort rule. Finally, we will derive some properties of the group optimal sharing rule relative to group size.

We first examine the chosen platform under an egalitarian sharing of the private good. Recalling the first order condition for a maximum, we have that λ_i^{ae} solves

$$\frac{du_i}{d\lambda_i} = \frac{dp_i}{d\lambda_i}\omega_i + p_i\frac{d\omega_i}{d\lambda_i} - \frac{dc}{d\lambda_i} = 0.$$

We can easily compute that

$$\frac{d\omega_i}{d\lambda_i} = b\left(v'(\lambda_i b) - \frac{1}{n_i}\right).$$

Since $v(.)$ is a strictly concave function and because of our assumption that $v'(b) \leq \frac{1}{n_G}$ it is immediate that there is a unique $\lambda_i^o \in (0,1]$ that maximizes $\omega(b, \lambda_i, n_i)$.

Clearly, when the win payoff ω_i is maximal so are the incentives to supply effort and hence the win probability. However, the group optimal platform also has to take into account the individual cost of supplying such effort. The following result shows that groups that are able to self-organize and use the egalitarian rule still would choose to maximize the equilibrium win payoff.

Proposition 2 *Under an egalitarian division the group optimal platform* λ_i^{ae} *maximizes the equilibrium win payoff. This platform is implicitly defined by*

$$n_i v^{'}(\lambda_i^{ae} b) = 1. \tag{24}$$

Proof See appendix.

Notice that λ_i^{ae} is independent of all the endogenous variables and hence independent of the platforms chosen by the other groups.

Let r_i^{ae} be the effort contributed by each individual under this platform. The expected equilibrium payoff to each group member will be

$$u_i^{ae} = \frac{n_i r_i^{ae}}{R_{i-} + n_i r_i^{ae}} \omega(\lambda_i^{ae}, n_i) - c(r_i^{ae}). \tag{25}$$

We shall now compare this utility with the utility of a representative group member under the relative effort rule u_i^{as}. The expected equilibrium payoff to each group member under the relative effort rule is

$$u_i^{as} = \frac{n_i r_i^{as}}{R_{i-} + n_i r_i^{as}} \omega(\lambda_i^{as}, n_i) - c(r_i^{as}) \tag{26}$$

where λ_i^{as} is the group optimal degree of publicness under the relative effort rule and r_i^{as} is the corresponding contributed effort.

Notice that in a second stage equilibrium all members of a group i contribute the same effort and hence each group member receives an equal share $\frac{1}{n_i}$ of the private good. Therefore for a given λ_i the equilibrium win payoff $\omega_i = v(\lambda_i b) + \frac{(1-\lambda_i)b}{n_i}$ is the same under the relative effort rule and under the egalitarian rule. Of course, the resources contributed under the two sharing rules will in general not be the same. This will allow us to compare u_i^{as} and u_i^{ae} without characterizing λ_i^{as}.

We have shown that λ_i^{ae} maximizes $\omega(.)$. Therefore,

$$\frac{n_i r_i^{as}}{R_{i-} + n_i r_i^{as}} \omega(\lambda_i^{ae}, n_i) - c(r_i^{as}) \geq u_i^{as}.$$

But since for λ_i^{ae} individuals choose $r_i^{ae} \neq r_i^{as}$, by revealed preference we have that

$$u_i^{ae} = \frac{n_i r_i^{ae}}{R_{i-} + n_i r_i^{ae}} \omega(\lambda_i^{ae}, n_i) - c(r_i^{ae}) > \frac{n_i r_i^{as}}{R_{i-} + n_i r_i^{as}} \omega(\lambda_i^{ae}, n_i) - c(r_i^{as}) \geq u_i^{as}.$$

The preference of equality over incentives is not conditioned by the platforms of the other groups. We thus have proven the following Proposition.

Proposition 3 *Irrespective of the platforms and the type of leadership adopted by the other groups, each group will choose the platform consisting of* λ_i^{ae} *and the egalitarian distribution of the private good.*

Proposition 3 has the remarkable implication that if groups were able to self-organize they would choose an egalitarian distribution of the private good even if efficient incentives to individual performance were available. It follows that the existence of incentives is a sign that groups were not able to self-organize. In the following section we will show that incentives are in the personal interest of entrepreneurial leaders. But before doing so we want to examine the overall group optimal platform in more detail. In particular, we are interested in how the degree of publicness and the win probability depend on group size. From expression (24) the following result is immediate.

Proposition 4 *The degree of publicness of the group optimal platform* λ_i^{ae} *is increasing in the group size,* n_i.

This proposition tells us that the larger the group the more "socially minded" will be their platform. Therefore, small groups will appear as greedier than large group. This platform choice also affects the win probabilities of the groups. The following proposition generalizes Esteban and Ray (2001)'s result.

Proposition 5 *With the group optimal platform the win probability p_i is increasing in the group size, n_i.*

Proof See appendix.

Larger groups choose a larger share of publicness in their platform and thereby mitigate the free-rider problem by reducing the effect of group size on the payoff of group members. This is why larger groups succeed in having higher win probabilities.

4.3 Entrepreneurial Leaders

Entrepreneurial leaders seek to maximize their win probability. We start by characterizing the chosen λ_i^{oh} under the two different sharing rules and then show that entrepreneurial leaders would opt for the introduction of incentives.

We first characterize the optimal platform by an external leader under an egalitarian division.

Proposition 6 *Under an egalitarian division the optimal platform chosen by an entrepreneurial leader coincides with the group-optimal platform. This platform λ_i^{oe} is implicitly defined by*

$$n_i v^{'}(\lambda_i^{oe} b) = 1. \tag{27}$$

Proof See appendix.

The intuition for this result is quite straightforward. The effort contributed by individuals is an increasing function of the payoff in case of victory. Hence, entrepreneurial leaders will choose the composition public/private that maximizes the value of ω_i. Notice that in the absence of individualized incentives leaders act as if they were concerned with the well-being of their constituency. Without incentives there would be no cost to entrepreneurial leadership. Our previous results for group decision apply here: the public good share and the win probability increase with group size.

Entrepreneurial leaders might do better by introducing incentives that link the amount of the private good received by each individual to her supply of effort. But now, the larger the share of the public good, the smaller the budget that can be used as incentives for collective action. We shall examine this trade-off and check whether large groups continue to be more "socially minded" and have higher win probabilities.

As a quick reminder, let us recall the implicit characterization of the optimal individual choice with incentives to effort. Transcribing (13) we have

$$\frac{1}{R}(1-p_i)\left(v(\lambda_i b)+\frac{(1-\lambda_i)b}{n_i}\right)+\frac{(1-\lambda_i)b(n_i-1)}{n_i R}-c'(\frac{p_i R}{n_i})=0.$$

This implicitly defines

$$p_i=\xi(R,\lambda_i,n_i).$$

The equilibrium R^* is obtained from the condition

$$\sum_j \xi(R^*,\lambda_j,n_j,b)=1.$$

which implicitly defines

$$R^*=\phi(\lambda,n).$$

Hence, the equilibrium win probabilities will be

$$p_i=\xi\left(\phi(\lambda,n),\lambda_i,n_i\right).$$

Entrepreneurial leaders will choose λ_i in order to maximize this equilibrium p_i. Accordingly with Remark 1

$$sign\frac{dp_i}{d\lambda_i}=sign\frac{\partial\xi(\lambda_i)}{\partial\lambda_i}.$$

Differentiating (13) we obtain

$$\begin{aligned}\frac{\partial\xi(\lambda_i)}{\partial\lambda_i} &= \frac{\frac{1}{R}(1-p_i)\left[bv'(\lambda_i b)-\frac{b}{n_i}\right]-\frac{b(n_i-1)}{Rn_i}}{\frac{1}{R}\left(v(\lambda_i b)+\frac{(1-\lambda_i)b}{n_i}\right)+\frac{R}{n_i}c''(\frac{p_i R}{n_i})}\\ &= \frac{\frac{(1-p_i)b}{Rn_i}\left[n_i v'(\lambda_i b)-\frac{n_i-p_i}{1-p_i}\right]}{\frac{1}{R}\left(v(\lambda_i b)+\frac{(1-\lambda_i)b}{n_i}\right)+\frac{R}{n_i}c''(\frac{p_i R}{n_i})}. \qquad (28)\end{aligned}$$

The sign of (28) depends on the sign of its numerator only. This is strictly decreasing in λ_i, strictly positive for $\lambda_i=0$ and strictly negative for $\lambda_i=1$. Hence, we have the following Proposition.

Proposition 7 *Let there be incentives to effort, then an entrepreneurial leader will choose the unique λ_i^{os} satisfying*

$$n_i v^{'}(\lambda_i^{os} b) = \frac{n_i - p_i}{1 - p_i} = 1 + \frac{n_i - 1}{1 - p_i}. \qquad (29)$$

Observe that the need for incentives does not make leaders precipitate complete privateness. However, as shown by the following proposition, the use of incentives by entrepreneurial leaders distorts platforms towards less publicness than under an egalitarian split.

Proposition 8 $\lambda_i^{os} < \lambda_i^{oe}$. *Furthermore, λ_i^{os} maximizes total effort / waste R.*

Proof See appendix.

Under the relative effort rule entrepreneurial leaders give too much weight to the private good relative to what is optimal for the group.

Let us now examine how this distortion affects the relationship between group size and the publicness of the platform and the win probability in a given equilibrium. To do this, consider the implicit characterization of the optimal platform chosen by an entrepreneurial leader as given by (29). In any equilibrium —and hence for R fixed— this equation has to hold for all groups, i.e. for all n_i. Therefore, we can obtain the relationship between publicness and group size by differentiating with respect to λ and n in (29) while holding R constant. We find that with individual incentives to effort entrepreneurial leaders choose platforms that still give larger groups a higher win probability.

Proposition 9 *If an entrepreneurial leader uses incentives to the supply of effort, then the win probability increases with group size and the degree of publicness of the platform decreases with size.*

Proof See appendix.

The efficiency of the larger groups is at the cost of reducing publicness. Notice that now it is thanks to an increased share of private goods that larger groups succeed to have higher win probabilities.

But would we observe incentives under entrepreneurial leadership? We shall now show that, unlike the case of group decision, entrepreneurial leaders prefer the use of incentives, irrespective of the platforms of the other groups.

Proposition 10 *Taking the behavior of the other leaders as fixed, all leaders can increase their win probability by introducing incentives.*

Proof See appendix.

We wish to remark two points here. First, leaders prefer to introduce incentives *irrespective of whether the other leaders are using incentives or not*. Secondly, note that moving from egalitarianism to incentives increases the win probability because it also increases the amount of effort contributed by the individual group members.[17] From Proposition 10 we directly have the following proposition.

Proposition 11 *Let entrepreneurial leaders be able to choose the platform, including the option between an egalitarian distribution of the private good or incentives. Then, there is a unique equilibrium in which all entrepreneurial leaders use the platform λ_i^{os} (hence with incentives).*

5 Individual Incentives and the Costs of Leadership

We now summarize the previous results to assess the cost of external leadership.

The first and basic point to stress is that entrepreneurial leaders can have a negative bias only through the use of incentives. If entrepreneurs were restricted to use the egalitarian sharing rule they would choose the group optimal platform. It is due to the use of incentives that the diverging goals of the leader and the community materialize and produce inefficient biases. In contrast, in group decision incentives are not used even if available in spite of the potentially beneficial effects of incentives on free-riding in the effort decision stage.[18]

[17]Of course, it must also decrease the win probability of other groups. We shall deal with this issue in the next section.

[18]Observe that we would get exactly the same result if we generalized our sharing rules of the private benefits to all possible linear combinations of the egalitarian and the relative effort rule. Our revealed preference argument that the egalitarian rule is group optimal would still hold. Also, entreprenerial leaders would go all the way towards the relative effort rule since the relative effort rule gives the highest weight to incentives and therefore maximizes win probabilities.

The contrast with the benchmark group-decision case reveals that leadership biases the platforms towards greater greediness (higher share of pocketable benefits) and greater resources expended (wasted) into trying to win command over the budget. Therefore individuals suffer the costs on two counts: (i) they don't obtain the public/private mix they would prefer and (ii) they expend more resources than what they would otherwise do.

With entrepreneurial leaders the bias towards privateness is increasing in group size. In contrast, with group-decision making privateness is decreasing in group size. It follows that the deviation from the original goals is larger the bigger is the group.[19]

The second type of bias concerns the over-expending of resources. The unilateral use of incentives will lead to higher win probabilities, but if all groups use incentives and hence spend more resources, it cannot be that all the win probabilities increase. The following proposition identifies who are the losers and who are the winners in terms of success probability resulting from the over-expending of resources.

Proposition 12 *Let the cost function have constant elasticity* $c(r) = \frac{1}{1+\eta} r^{1+\eta}$ *and let* $v(.)$ *be sufficiently close to linearity. There is a threshold level in group size such that the groups with a smaller size will have a lower win probability in the equilibrium with incentives than with an egalitarian distribution and all groups with a larger size will have a higher win probability.*

Proof See appendix.

From this Proposition it follows that if the economy were in an equilibrium with egalitarianism and leaders could compare equilibria, it would be the leaders of the larger groups the ones that would have an interest in unfolding a process of introduction of incentives.[20]

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[19] Going back to the discussion on German political parties by Michels, our results imply that the SPD - much larger than the communist party - would appear as "betraying" the ideals far more intensely than the communists.

[20] The case of higher degrees of concavity in $v(.)$ remains to be studied. It seems plausible that for high degrees of concavity the result gets reversed and the smaller groups are the main beneficiaries.

To see why observe that there are two counter-vailing forces at work. Incentives eliminate free-rider problems because individuals are rewarded by what they contribute only. On the one hand, free-riding is more severe in larger groups since an individuals deviation has a smaller effect on the win probabilities in bigger groups. Hence larger groups benefit

6 Conclusion

This paper studies how leader's use the group platform to provide incentives for collective action and the cost of external leadership. Collective action organizations take many different forms and therefore also different leadership structures. In this paper we have concentrated on two extreme forms: group optimal decision making which implies a decentralized democratic structure and entrepreneurial leaders with central power. Given the voluntary nature of participation in collective action it seems reasonable that leaders will have to respect at least the general goals of their group members. Hence, the distortion introduced by entrepreneurial leaders seems the maximal distortion possible.[21] To conclude, let us now contrast our results with the available evidence.

If the predictions of our model were correct, we should find disagreement among group members and their leaders concerning the relative importance of the different goals of the organization. In particular leaders should evaluate goals with private benefits relatively more than group members. Knoke (1990) has studied this issue using the National Association Study (NSA).[22] Both leaders and members were asked their perceptions of the importance of the different incentives (group goals). Knoke (1990) studies the degree of consistency between leaders' and members' evaluations by correlating the importance ratings over 14 goals/incentives across 35 associations. For five categories no significant covariance is found, suggesting that leaders incorrectly perceived their importance for their membership, namely, research, information and data services, organizational negotiations, organizational general prestige and emphasizing the main goals and purposes of the group. The first four categories clearly fall into the group specific public good category, suggesting in line with our prediction that leaders underestimate the

more from the elimination of free-riding. On the other hand, smaller groups can pay higher per unit incentive rates than larger groups. Which effect dominates will depend on the concavity of $v(.)$.

[21] One way to include intermediate leadership structures in our context would be by letting leaders to be partially motivated by success and partially by the well-being of the group members. Given the assumptions of our model, leaders would maximize $\beta p_i + (1 - \beta) u_i$ where β tells us the relative importance of the success motive.

[22] The NSA draws representative samples of 459 collective action organizations from the population of 13,000 national assocations and 8746 members from 35 professional, recreational and women's organizations. For details on the NSA see Chapter 4 in Knoke (1990).

importance of these categories for group members.

By seeking success entrepreneurial leaders deviate from an egalitarian split and from the group optimal platform. Hence, we should expect that entrepreneurial leaders might not be perceived as a valid spokesman by the individual members of the group. Gamson (1975,1980,1990) finds support for this. He divides a group's notion of success into two clusters. One cluster concerns the acceptance of the group's leader by its members and the other cluster focuses on the achievements (whether the group's beneficiary gains new advantages). Gamson then proceeds to analyze the success / failure of 53 randomly selected movement organizations active in the United States between 1980 and 1945. "Centralization" was used as a label for groups with a dominant leader and "bureaucracy" was used as a label for groups with a written constitution, formal lists of members and that possessed at least three distinct levels of internal divisions. Gamson (1980) shows that bureaucracy is associated with acceptance while centralization is associated with new advantages. These findings give support to our assumption and results for entrepreneurial leaders: they maximize success (achieve new advantages) but loose on acceptance (diverge from the group optimal platform).

The present paper takes the leadership structure as given. What makes some groups able to reach decisions while others simply follow a leader is an open question. Group size has obviously a role to play[23] and so does the importance of the issue at stake. Barakso and Schaffner (2008) provide some evidence based on a random sample of 114 national membership organizations in the US that larger organizations tend to be less democratic than smaller organizations. Moreover they link the organizational structure to exit costs and argue in line with the evidence that groups with higher exit costs, in particular professional associations and unions, are structured more democratically. These exit costs are the benefits lost when leaving the group. Many of these benefits have a group specific public good character, e.g. access to professional journals. Seen this way, the evidence suggests that groups with a high group specific public component are more democratic. This is in line with what our model predicts, except that the causality is reversed: in our model it is because these groups are more democratic that they give more weight to the public good component.

[23]But in classical Greece the assembly of citizens, the ultimate decision body, could gather well over 10,000 participants.

7 Appendix

Lemma 1 For each group i there is a unique r_i that satisfies (9) for each individual k.
Proof. For the equalitarian case (9) becomes

$$\frac{1-p_i}{R}\left(v(\lambda_i b)+\frac{(1-\lambda_i)b}{n_i}\right)-c'(r_{ik})=0$$

It is immediate that this equation has a unique solution $r_{ik}=r_i>0$ for all individuals k of group i.

For the case with incentives under the relative effort rule we have that

$$p_i\frac{\partial\omega_{ik}}{\partial r_{ik}}=p_i\frac{(1-\lambda_i)\,b(R_i-r_{ik})}{R_i^2}=\frac{(1-\lambda_i)\,b}{R}(1-\frac{r_{ik}}{R_i}).$$

Using this fact in (9) and rearranging we obtain

$$\frac{\partial u_{ik}}{\partial r_{ik}}=\frac{1}{R}\big((1-p_i)v(\lambda_i g)+(1-\lambda_i)b\big)-\big(\frac{(1-\lambda_i)b)}{R^2}r_{ik}+c'(r_{ik})\big).$$

Notice that the first brackets is common to all k of group i and that the second brackets is strictly increasing in r_{ik}. Therefore, there is a unique value $r_{ik}=r_i=\frac{R_i}{n_i}$ that solves $\frac{\partial u_{ik}}{\partial r_{ik}}=0$. ■

Lemma 2 *In each case —egalitarian and incentives— the unique r_i maximizing u_i is implicitly given by*

$$\frac{1-p_i}{R}\left(v(\lambda_i b)+\frac{(1-\lambda_i)b}{n_i}\right)-c'(r_i)=0(egalitarian),or \tag{30}$$

$$\frac{(1-p_i)}{R}(v(\lambda_i g)+\frac{(1-\lambda_i)\,br_i}{R_i})+p_i\frac{(1-\lambda_i)\,b(R_i-r_i)}{R_i^2}-c'(r_i)=0(incentives). \tag{31}$$

Proof. Differentiating (7) with respect to r_i we obtain the first order condition

$$\frac{\partial u_i}{\partial r_i}=\frac{1-p_i}{R}\omega_i+p_i\frac{\partial\omega_i}{\partial r_i}-c'(r_i)=0. \tag{32}$$

We start by noticing that under an egalitarian distribution $\frac{\partial\omega_i}{\partial r_i}=\frac{\partial^2\omega_i}{\partial^2 r_i}=0$, with incentives $\frac{\partial\omega_i}{\partial r_i}=\frac{(1-\lambda_i)b(R_i-r_i)}{R_i^2}$ and $\frac{\partial^2\omega_i}{\partial^2 r_i}=-\frac{2(1-\lambda_i)b(R_i-r_i)}{R_i^3}$.

Observe now that $\frac{\partial u_i}{\partial r_i}$ is continuous in r_i for $R > 0$ and that $\lim_{r_i \to 0} \frac{\partial u_i}{\partial r_i} = \frac{\omega_i}{R} > 0$ and that $\lim_{r_i \to \infty} \frac{\partial u_i}{\partial r_i} = -\infty$. Therefore, there must exist at least one $r_i > 0$ for which $\frac{\partial u_i}{\partial r_i} = 0$. Suppose now that $r_i = 0$ and $R = 0$ so that $p_i = \frac{n_i}{n} < 1$. By taking an arbitrarily small $r_i > 0$ we would make $p_i = 1$. Hence in no case $r_i = 0$ can be a best reply for an individual of type i.

We shall now show that there is a unique $r_i > 0$ satisfying the first order condition $\frac{\partial u_i}{\partial r_i} = 0$ and that this condition indeed identifies a maximum.

Differentiating (32) with respect to r_i we obtain

$$\frac{\partial^2 u_i}{\partial r_i^2} = -2\frac{1-p_i}{R^2}\omega_i + 2\frac{1-p_i}{R}\frac{\partial \omega_i}{\partial r_i} + p_i\frac{\partial^2 \omega_i}{\partial r_i^2} - c''(r_i).$$

We can immediately deduce that under egalitarianism u_i is strictly concave in r_i.

For the case with incentives this second derivative becomes

$$\begin{aligned}\frac{\partial^2 u_i}{\partial r_i^2} &= -\frac{2}{R^2}(1-p_i)\left(v(\lambda_i b) + \frac{(1-\lambda_i)\, b r_i}{R_i}\right) + \frac{2}{R}(1-p_i)\frac{(1-\lambda_i)\, b(R_i - r_i)}{R_i^2} \\ &\quad -p_i\frac{2\,(1-\lambda_i)\, b(R_i - r_i)}{R_i^3} - c''(r_i)\end{aligned}$$

Since the first and the third term on the right hand side are negative, if we can show that the second term plus the fourth term are negative, we are done. Hence, we need to look at

$$\frac{2}{R}(1-p_i)\frac{(1-\lambda_i)\, b(R_i - r_i)}{R_i^2} - c''(r_i)$$

Using now (31) we obtain

$$\begin{aligned}&\frac{2}{R}(1-p_i)\frac{(1-\lambda_i)\, b(R_i - r_i)}{R_i^2} - c''(r_i) \\ = \;& \frac{2}{R}(1-p_i)\left[\frac{c'(r_i)}{p_i} - \frac{1}{R}\frac{(1-p_i)}{p_i}\left(v(\lambda_i b) + \frac{(1-\lambda_i)\, b r_i}{R_i}\right)\right] - c''(r_i) \\ = \;& \frac{2}{R}(1-p_i)\frac{c'(r_i)}{p_i} - c''(r_i) - \frac{2}{R^2}\frac{(1-p_i)^2}{p_i}\left(v(\lambda_i b) + \frac{(1-\lambda_i)\, b r_i}{R_i}\right)\end{aligned}$$

If we can show that $\frac{2}{R}(1-p_i)\frac{c'(r_i)}{p_i} - c''(r_i)$ is negative we are done.

$$\begin{aligned} & \frac{2}{R}(1-p_i)\frac{c'(r_i)}{p_i} - c''(r_i) \\ = & \frac{c'(r_i)}{r_i}\left[\frac{2(1-p_i)r_i}{Rp_i} - \eta(r_i)\right] \\ = & \frac{c'(r_i)}{r_i}\left[\frac{2(1-p_i)}{n_i} - \eta(r_i)\right] \end{aligned}$$

since $p_i = \frac{n_i r_i}{R}$ in a symmetric equilibrium. But $\frac{2(1-p_i)}{n_i} - \eta(r_i) < 0$ since $n_i \geq 2$ and $\eta(r_i) \geq 1$. Therefore $\frac{\partial^2 u_i}{\partial r_i^2} < 0$. ■

LEMMA 3 $p_i = \psi(R, \lambda_i, n_i, g)$ *is a continuous strictly decreasing function of* R *for all* i.

Proof. Using the implicit definition of p_i in (12) and (13) for the egalitarian and incentives rule we observe that ψ is continuous because both v and c' are continuous. Furthermore, differentiating with respect to R we have:

$$\frac{\partial p_i}{\partial R} = -\frac{\frac{(1-p_i)\omega_i}{R^2} + \frac{p_i c''}{n_j}}{\frac{\omega_i}{R} + \frac{Rc''}{n_i}} < 0 \text{ (egalitarian).}$$

$$\frac{\partial p_i}{\partial R} = -\frac{\frac{1}{R^2}(1-p_i)\left(v(\lambda_i b) + \frac{(1-\lambda_i)b}{n_i}\right) + \frac{(1-\lambda_i)b(n_i-1)}{R^2} + \frac{p_i}{n_i}c''(\frac{p_i R}{n_i})}{\frac{1}{R}\left(v(\lambda_i b) + \frac{(1-\lambda_i)b}{n_i}\right) + \frac{R}{n_i}c''(\frac{p_i R}{n_i})} < 0 \text{ (incentives)}$$

■

PROPOSITION 1 *For every set of parameters* (λ, n, g) *there exists an equilibrium and it is unique.*

Proof. We have already seen that for each R there is a unique vector of win probabilities defined by (15). The only point that remains to be proven is that there is a unique value of R satisfying (16). By Lemma 3 p_i is a continuous strictly decreasing function of R for all i.

An inspection of the first order conditions (12) and (13) for the egalitarian and the incentives case, respectively, immediately reveals that in both $p \to 1$ as $R \to 0$ and $p \to 0$ as $R \to \infty$. Hence there is a unique R^* satisfying the equilibrium condition (16). ■

PROPOSITION 2 Under an egalitarian division the group optimal platform λ_i^{ae} maximizes the equilibrium win payoff. This payoff is implicitly defined by

$$n_i v^{'}(\lambda_i^{ae} b) = 1.$$

Proof. Recall that the first order condition (21) is given by

$$0 = \left(\omega_i - \frac{R}{n_i} c'(p_i \frac{R}{n_i})\right) \frac{\partial p_i}{\partial \lambda_i} + p_i \frac{\partial \omega_i}{\partial \lambda_i} - \frac{p_i}{n_i} c' \left(p_i \frac{R}{n_i}\right) \frac{\partial R}{\partial \lambda_i}. \quad (33)$$

Now replace $c'\left(p_i \frac{R}{n_i}\right)$ using (9) and use equation (22) for $\frac{\partial R}{\partial \lambda_i}$ and equation (23) for $\frac{\partial p_i}{\partial \lambda_i}$.

Then, the first order condition becomes

$$0 = \frac{\partial \omega_i}{\partial \lambda_i} \left(\frac{\partial \psi_i}{\partial \omega_i} \left(1 - \frac{\frac{\partial \psi_i}{\partial R_i}}{\sum_j \frac{\partial \psi_j}{\partial R}}\right) \omega_i \left(1 - \frac{1 - p_i}{n_i}\right) + p_i + \frac{p_i}{R} \frac{1 - p_i}{n_i} \omega_i \frac{\frac{\partial \psi_i}{\partial \omega_i}}{\sum_j \frac{\partial \psi_j}{\partial R}} \right)$$

Since $\frac{\partial \psi_i}{\partial \omega_i} > 0$ and $\frac{\partial \psi_j}{\partial R} < 0 \ \forall j$, the first term of the sum in the big braces is positive.

For later use we calculate $\frac{\partial \psi_i}{\partial \omega_i}$ and $\frac{\partial \psi_i}{\partial R}$ explicitly:

$$\frac{\partial \psi_i}{\partial \omega_i} = \frac{\frac{1-p_i}{R}}{\frac{\omega_i}{R} + \frac{Rc''}{n_i}} \quad (34)$$

$$\frac{\partial \psi_i}{\partial R} = -\frac{\frac{(1-p_i)\omega_i}{R^2} + \frac{p_i c''}{n_i}}{\frac{\omega_i}{R} + \frac{Rc''}{n_i}} < 0. \quad (35)$$

We now show that the sum of the second and third term is also positive. Dividing by p_i and rearranging this is equivalent to showing that

$$\frac{1 - p_i}{n_i R} \omega_i \frac{\partial \psi_i}{\partial \omega_i} < - \sum_j \frac{\partial \psi_j}{\partial R}$$

After introducing (34) and (35) and rearranging we see that clearly

$$\frac{1 - p_i}{n_i} \frac{\frac{(1-p_i)\omega_i}{R^2}}{\frac{\omega_i}{R} + \frac{Rc''}{n_i}} < \sum_j \frac{\frac{(1-p_j)\omega_j}{R^2}}{\frac{\omega_j}{R} + \frac{Rc''}{n_j}} + \sum_j \frac{\frac{p_j c''}{n_j}}{\frac{\omega_j}{R} + \frac{Rc''}{n_j}}.$$

Thus, the sign of $\frac{du_i}{d\lambda_i}$ is the same as the sign of $\frac{\partial \omega_i}{\partial \lambda_i}$.

Recall that

$$\omega_i = v(\lambda_i b) + (1-\lambda)\frac{b}{n_i}.$$

Since $v(.)$ is concave, it follows that $\omega_i(.)$ is also concave. By differentiation we find that the maximum is attained for $\lambda_i = \lambda_i^{ae}$, as defined in this Proposition. ■

PROPOSITION 5 With the group optimal platform the win probability p_i is increasing in the group size, n_i.
Proof. We can write

$$\frac{dp_i}{dn_i} = \frac{dp_i}{d\lambda_i}\frac{\partial \lambda_i}{\partial n_i} + \frac{\partial p_i}{\partial n_i}.$$

In Proposition 6 we will prove that λ_i^{ae} maximizes p_i. Hence, the above expression becomes

$$\frac{dp_i}{dn_i} = \frac{\partial p_i}{\partial n_i}.$$

Partially differentiating in (12) we obtain

$$\frac{\partial p_i}{\partial n_i} = \frac{Rr_i c^{''}(r_i) - \frac{(1-p_i)(1-\lambda_i)b}{n_i}}{n_i\omega_i + R^2 c''(r_i)}.$$

Using (12) we finally obtain

$$\frac{dp_i}{dn} = \frac{Rc^{'}(r_i)\left(\eta(r_i) - 1\right) + (1-p_i)v(\lambda_i g)}{n_i\omega_i + R^2 c''(r_i)} > 0.$$

■

PROPOSITION 6 Under an egalitarian division the optimal platform chosen by an entrepreneurial leader coincides with the group-optimal platform. This platform λ_i^{oe} is implicitly defined by

$$n_i v^{'}(\lambda_i^{oe} b) = 1. \tag{36}$$

Proof. From Lemma 1 we have that the sign of $\frac{dp_i}{d\lambda_i}$is the same as the sign of $\frac{\partial \psi_i}{\partial \lambda_i}$.

In view of (13) and of (15) we can write

$$\frac{\partial \psi_i}{\partial \lambda_i} = \frac{\partial \psi_i}{\partial \omega_i} \frac{\partial \omega_i}{\partial \lambda_i}.$$

We have seen that $\frac{\partial \psi_i}{\partial \omega_i} > 0$. Therefore, p_i is maximal when ω_i is maximal. In Proposition 2 we have demonstrated that this maximum is attained for $\lambda_i^{ae} = \lambda_i^{oe}$. ■

PROPOSITION 8 $\lambda_i^o < \lambda_i^e$. Furthermore, λ_i^o maximizes waste R.

Proof. Recall that λ_i^e satisfies $n_i v'(\lambda_i^e b) = 1$. Let's look again at (28). Notice that

$$n_i v'(\lambda_i b) \leq 1 < \frac{n_i - 1}{1 - p_i} \text{ for } \forall \lambda_i \geq \lambda_i^e$$

Hence, we have

$$\frac{\partial \xi(\lambda_i)}{\partial \lambda_i} = \frac{\frac{(1-p_i)b}{Rn_i}\left[n_i v'(\lambda_i b) - \frac{n_i - p_i}{1-p_i}\right]}{\frac{1}{R}\left(v(\lambda_i b) + \frac{(1-\lambda_i)b}{n_i}\right) + \frac{R}{n_i} c''(\frac{p_i R}{n_i})} < 0 \text{ for } \forall \lambda_i \geq \lambda_i^e$$

Therefore $\lambda_i^o < \lambda_i^e$. To see that λ_i^o maximizes waste R recall that by remark 2 R attains a maximum with respect to λ_i when p_i is maximal, i.e. at λ_i^o. ■

PROPOSITION 9 *If an entrepreneurial leader uses incentives to the supply of effort, then the win probability increases with group size and the degree of publicness of the platform decreases with size.*

Proof. We start by performing the differentiation with respect to λ and n in (29). Rearranging we obtain,

$$\frac{d\lambda_i}{dn_i} = -\frac{(1-p_i)^2 v'(\lambda_i b) - \left[(1-p_i) + (n_i - 1)\frac{\partial \xi(\lambda_i)}{\partial n_i}\right]}{(1-p_i)^2 b n_i v''(\lambda_i b)}. \tag{37}$$

Notice that because of the concavity of v the denominator is negative. Hence, the sign of the derivative depends on the sign of the numerator and this in turn depends on the sign and size of $\frac{\partial \xi(\lambda_i)}{\partial n_i}$. We turn now to this.

Let us now partially differentiate p_i with respect to n_i in the first order condition for individual choice to obtain

$$
\begin{aligned}
\frac{\partial \xi_i}{\partial n_i} &= \frac{-\frac{1}{R}(1-p_i)\frac{(1-\lambda_i)b}{n_i^2} + \frac{(1-\lambda_i)b}{R}\left(\frac{n_i-(n_i-1)}{n_i^2}\right) + \frac{p_i R}{n_i^2}c''(\frac{p_i R}{n_i})}{\frac{1}{R}\left(v(\lambda_i b) + \frac{(1-\lambda_i)b}{n_i}\right) + \frac{R}{n_i}c''(\frac{p_i R}{n_i})} \\
&= \frac{\frac{(1-\lambda_i)b}{R n_i^2}p_i + \frac{p_i R}{n_i^2}c''(\frac{p_i R}{n_i})}{\frac{1}{R}\left(v(\lambda_i b) + \frac{(1-\lambda_i)b}{n_i}\right) + \frac{R}{n_i}c''(\frac{p_i R}{n_i})} > 0.
\end{aligned}
$$

So the sign of $\frac{d\lambda_i}{dn_i}$ depends on the sign of

$$
\begin{aligned}
& (1-p_i)^2 v'(\lambda_i b) - \left[(1-p_i) + (n_i - 1)\frac{\partial \xi(\lambda_i)}{\partial n_i}\right] \\
=\ & (1-p_i)\left[(1-p_i)v'(\lambda_i b) - 1\right] - \left[(n_i-1)\frac{\partial \xi(\lambda_i)}{\partial n_i}\right] \\
=\ & (1-p_i)\left[\frac{n_i - p_i}{n_i} - 1\right] - \left[(n_i-1)\frac{\partial \xi(\lambda_i)}{\partial n_i}\right]
\end{aligned}
$$

since in the optimal platform $(1-p_i)v'(\lambda_i b) = \frac{n_i - p_i}{n_i}$. But clearly $\left[\frac{n_i-p_i}{n_i} - 1\right] < 0$, so the sign of $\frac{d\lambda_i}{dn_i}$ is negative. ■

PROPOSITION 10 *Taking the behavior of the other leaders as fixed, all leaders can increase their win probability by introducing incentives.*

Proof. We shall show that even holding the platform λ_i constant the leader will increase the win probability when introducing incentives.

Notice first that holding λ_i constant the equilibrium win payoff will be the same in the two cases, ω_i.

The first order condition for individual effort in the case of incentives is

$$
\frac{1-p_i}{R}\omega_i + \delta_i = c'(p_i \frac{R}{n_i}),
$$

where $\delta_i \equiv \frac{(1-\lambda_i)b(n_i-1)}{n_i R} > 0$. Notice, further that the first order condition under the egalitarian distribution is the same taking $\delta_i = 0$.

Hence, the result simply follows from the fact that the equilibrium p_i is strictly increasing in δ_i. This can be easily obtained from differentiation and following the same steps as in previous Propositions. ■

PROPOSITION 12 *For constant elasticity cost functions $c(r) = \frac{1}{1+\eta}r^{1+\eta}$ and $v(.)$ sufficiently close to linearity, if we compare the equilibrium with* *incentives with that with egalitarianism there is a threshold level of groups size such that the groups with a smaller size will have a higher win probability under egalitarianism and all groups with a larger size will have a lower win probability.*

Proof. We shall compute the first order conditions for the two equilibria. We shall use the superindices e and s to denote the equilibrium values under egalitarianism and under incentives respectively. For a constant elasticity cost function the first order conditions become

$$\begin{aligned} \frac{1}{R^e}(1-p_j^e)\omega_j^e &= \left(\frac{p_j^e R^e}{n_j}\right)^\eta \text{ and } n_j v'(\lambda_j^e b) = 1 \\ \frac{1}{R^s}(1-p_j^s)\omega_j^s + \frac{(1-\lambda_i)b(n_j-1)}{n_j R} &= \left(\frac{p_j^s R^s}{n_j}\right)^\eta \text{ and } n_j v'(\lambda_j^s b) = \frac{n_j - p_j}{1-p_j^s} \end{aligned}$$

Rearranging and dividing the first order condition for individuals

$$\begin{aligned} \left(\frac{R^s}{R^e}\right)^{\eta+1} &= \frac{\frac{1-p_j^s}{p_j^{s\eta}}}{\frac{1-p_j^e}{p_j^{e\eta}}}\frac{\omega_j^s}{\omega_j^e} + \frac{\frac{(1-\lambda_j^s)b}{p_j^{s\eta}}\frac{n_j-1}{n_j}}{\frac{1-p_j^e}{p_j^{e\eta}}\omega_j^e} \\ &= \frac{f(p_j^s)}{f(p_j^e)}\frac{\omega_j^s + \frac{(1-\lambda_j^s)b}{1-p_j^s}\frac{n_j-1}{n_j}}{\omega_j^e} \\ &= \frac{f(p_j^s)}{f(p_j^e)}\frac{v(\lambda_j^s b) + \frac{n_j-p_j}{1-p_j^s}\frac{(1-\lambda_j^s)b}{n_j}}{v(\lambda_j^e b) + \frac{(1-\lambda_j^e)b}{n_j}} \end{aligned}$$

Introducing the FOC of the leaders we get

$$\left(\frac{R^s}{R^e}\right)^{\eta+1} = \frac{f(p_j^s)}{f(p_j^e)}\frac{v(\lambda_j^s b) + (1-\lambda_j^s)bv'(\lambda_j^s b)}{v(\lambda_j^e b) + (1-\lambda_j^e)bv'(\lambda_j^e b)}$$

Assume that $v(x) = \frac{1}{1-\alpha}x^{1-\alpha}$. Hence $v'(x) = x^{-\alpha}$ and $v(x) = \frac{xv'(x)}{1-\alpha}$. With this v-function we get

$$\begin{aligned} \left(\frac{R^s}{R^e}\right)^{\eta+1} &= \frac{f(p_j^s)}{f(p_j^e)}\frac{\lambda_j^s + (1-\lambda_j^s)(1-\alpha)}{\lambda_j^e + (1-\lambda_j^e)(1-\alpha)}\frac{bv'(\lambda_j^s g)}{bv'(\lambda_j^e g)} \\ &= \frac{f(p_j^s)}{f(p_j^e)}\frac{\lambda_j^s + (1-\lambda_j^s)(1-\alpha)}{\lambda_j^e + (1-\lambda_j^e)(1-\alpha)}\left(1 + \frac{n_j-1}{1-p_j^s}\right) \end{aligned}$$

We assume that $\frac{\lambda_j^s+(1-\lambda_j^s)(1-\alpha)}{\lambda_j^e+(1-\lambda_j^e)(1-\alpha)} \simeq \frac{\lambda_k^s+(1-\lambda_k^s)(1-\alpha)}{\lambda_k^e+(1-\lambda_k^e)(1-\alpha)}$. Note that if $\alpha = 0$ (i.e. v linear) $\frac{\lambda_j^s+(1-\lambda_j^s)(1-\alpha)}{\lambda_j^e+(1-\lambda_j^e)(1-\alpha)} = 1 \forall j$. Hence we have that

$$\frac{f(p_j^s)}{f(p_j^e)} K \left(1 + \frac{n_j - 1}{1 - p_j^s}\right) = \frac{f(p_k^s)}{f(p_k^e)} K \left(1 + \frac{n_k - 1}{1 - p_k^s}\right)$$

If win probabilities differ when we move from no incentives to incentives there must be at least one group j with $p_j^e > p_j^s$. If there is more than one such group, take the largest one. Since $f(p)$ is decreasing in p we have that $\frac{f(p_j^s)}{f(p_j^e)} > 1$. Since p is increasing in group size we have for $n_k < n_j$

$$1 + \frac{n_k - 1}{1 - p_k^s} < 1 + \frac{n_j - 1}{1 - p_j^s}$$

Therefore for all $n_k < n_j$

$$\frac{f(p_k^s)}{f(p_k^e)} > \frac{f(p_j^s)}{f(p_j^e)} > 1 \Rightarrow p_k^e > p_k^s$$

By construction there is no group larger than n_j for which this inequality holds true.

Similarly, there must be at least one group l such that $p_l^e < p_l^s$. If there is more than one such group take the smallest group. Since $f(p)$ is decreasing in p we now have that we have that $\frac{f(p_l^s)}{f(p_l^e)} < 1$. Since p is increasing in group size we have for $n_h > n_l$

$$1 + \frac{n_h - 1}{1 - p_h^s} > 1 + \frac{n_l - 1}{1 - p_l^s}$$

Therefore for all $n_h > n_l$

$$\frac{f(p_h^s)}{f(p_h^e)} < \frac{f(p_l^s)}{f(p_l^e)} < 1 \Rightarrow p_h^e < p_h^s$$

Consequently, there is a partition of the groups by a threshold size such that their win probabilities with incentives increases or decreases relative to the equilibrium with egalitarian incentives. ■

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