Factor decomposition of cross-country income inequality, 1960–1990

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Abstract

In this paper we present Esteban's 1994 [Esteban, J., 1994. La desigualdad interregional en Europa y en España: descripción y análisis. In: Esteban, J.M[a]., Vives, X. (dirs.), Crecimiento y convergencia regional en España y en Europa, Vol. 2, Instituto de Análisis Económico] decomposition of the Theil index of inequality over per capita incomes into the (unweighted) sum of the inequality indices of (i) the productivity per employed worker, (ii) the employment rate, (iii) the active over working-age population rate, and (iv) the working-age over total population rate. Each of these factors clearly have different meanings for analysis as well as for policy. We apply this factoral decomposition to a set of 120 countries. We also contrast the empirical findings with the results obtained for the 23 OECD countries. [OECD, Labour Force Statistics, Several issues, Paris]. © 1998 Elsevier Science S.A. All rights reserved.

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1. Introduction

Observed differences in national per capita incomes might be partially attributable to factors other than productivity differentials. Differences in participation and/or unemployment rates can induce differences in per capita income between countries or regions even when the average productivities per worker are equal. Carlino (1992) and Browne (1989), for instance, find that most of the computed interregional inequality in the US can be explained by the regional differences in the employment/population ratio.

This argument can be taken further. Per capita income x can be expressed as the product of the productivity per employed worker, y, times the employment rate, e, times the active over working-age population rate, a, times the working-age over total population rate, w. National inequalities in each of these factors clearly have different meanings for analysis as well as for policy.

In this paper we present Esteban's (1994) decomposition of the Theil index of inequality over per capita incomes into the (unweighted) sum of the indices corresponding to each of these factors and

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apply this decomposition to a set of 120 countries. We also contrast the empirical findings with the results obtained for the 23 OECD countries.

2. Decomposing income inequality by factors

Let x^i be the per capita income of country i, that is $x^i = X^i / N^i$ where X^i is the GDP and N^i total population. Let us now consider the widely used Theil index[1]

$$T(x, p) = \sum_i p^i \ln\left(\frac{\mu}{x^i}\right).$$ (1)

where p^i stands for the share of country i in the world population and μ for the world average per capita income.

Let E^i, A^i, and W^i be region's i total employment, active, and working-age population, respectively. Let $X, E, A, W,$ and P be the corresponding world values.

We shall denote $y^i = (X^i)/(E^i)$, $e^i = (E^i)/(A^i)$, $a^i = (A^i)/(W^i)$, and $w^i = (W^i)/(N^i)$. Clearly,

$$x^i \equiv y^i \times e^i \times a^i \times w^i.$$ (2)

We wish now to measure the contribution of each individual factor to measured inequality of per capita incomes. In order to do so, we shall define four fictitious regional incomes by letting regional specific factor values to differ from the average by only one at a time. Specifically we define fictitious incomes for region i, x_i^r, $r = y, e, a, w$, as follows

$$x_i^y = y_i \times e \times a \times w$$

$$x_i^e = y \times e_i \times a \times w$$

$$x_i^a = y \times e \times a_i \times w$$ (3)

$$x_i^w = y \times e \times a \times w_i,$$

where y, e, a and w are the world averages.

Let us now compute the degree of inequality with respect to μ that we would observe had regional incomes been x_i^r, $r = y, e, a, w$. Using the Theil index we have

$$T^y = \sum_i p_i \log\left(\frac{\mu}{x_i^y}\right) = \sum_i p_i \log\left(\frac{\mu}{y_i \times e \times a \times w}\right) = \sum_i p_i \log\left(\frac{y}{y_i}\right),$$ (4)

and similarly for e, a i w.

Adding over these inequality indices we obtain

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[1]See recent applications by Ram (1979), (1984), (1987) and Ram (1992); Theil (1989), and Theil and Deepak (1994). For the useful properties of the Theil index see, for instance, Bourguignon (1979) and Chakravarty (1990).

$$T^y + T^e + T^a + T^w = \sum_i p_i \left\{ \log\left(\frac{\mu}{x_i^y}\right) + \log\left(\frac{\mu}{x_i^e}\right) + \log\left(\frac{\mu}{x_i^a}\right) + \log\left(\frac{\mu}{x_i^w}\right) \right\} = \tag{5}$$

$$= \sum_i p_i \log\left(\frac{\mu^4}{x_i^y \times x_i^e \times x_i^a \times x_i^w}\right) = \sum_i p_i \log\left(\frac{\mu}{x_i}\right) = I \, ,$$

where I is the Theil index over the original per capita incomes[2].

Expression (5) decomposes the inequality of income into the sum of the inequalities displayed by four factors: productivity per worker, employment rate, activity rate over potentially active population and working-age population rate[3].This particular decomposition is interesting because each of these factors is governed by different forces. Factor mobility and competition are relevant for the eventual convergence in productivity per worker, but might not be critical for the other factors. International inequality in unemployment rates essentially depends on the importance of region-specific shocks. Increases in the international specialization following the growing world trade would tend to raise the importance of local shocks and thus increase the role played by this factor in total recorded inequality. International differences in activity rates, while influenced by the level of economic development and the existence of unemployment benefits, seem to depend on a variety of idiosyncratic sociological and cultural variables. Finally, the working-age population ratio is essentially explained by the national specific demographic patterns which, in turn, might depend on economic as well as sociological factors. Each of these factors clearly calls for a differentiated analysis.

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[2]Let us point out that in the computation of each factor inequality index we have used μ as reference income, rather than the corresponding mean of the relevant fictitious income x_i^r when weighted by the p_i, as it would correspond to a Theil index proper. Let μ^r be the weighted mean of x_i^r, that is,

$$\mu^r = \sum_i p_i x_i^r \, , \text{ for } r = y, e, a, w \, .$$

The Theil index would then be

$$I(p, x^r) = \sum_i p_i \log \frac{\mu^r}{x_i^r} \, .$$

In general there will be differences between $I(p, x^r)$ and $T^r = T(p, x^r)$. It is easy to obtain that

$$T(p, x^r) = I(p, x^r) + \log(\mu/\mu^r) \, .$$

Finally, for the Theil index of income inequality we have

$$I(p, x) = T(p, x^y) + T(p, x^e) + T(p, x^a) + T(p, x^w) =$$
$$= I(p, x^y) + I(p, x^e) + I(p, x^a) + I(p, x^w) + \{4 \log(\mu) - \log(\mu^y \mu^e \mu^a \mu^w)\} \, .$$

[3]The approach we have followed here is inspired in the *shift-share analysis* commonly used in regional economics. There too regional growth is decomposed in a number of factors by letting each region to differ from national average in one characteristic only at a time (typically sectoral structure and sector growth), as in Dunn (1960) and Esteban (1972).

3. Cross-country income inequality by factors

For our empirical work we shall use the Summers and Heston (1994) data set covering 120 countries for the period 1960–1989. Unfortunately, the information available is not sufficient to perform the factor decomposition in full. Data permit only to separate out average income per active population and active over total population. Thus, instead of (3), we shall be using

$$T(x, p) = T(yve, p) + T(avw, p) = \{T(y, p) + T(e, p)\} + \{T(a, p) + T(w, p)\}.$$

The results reported in Table 1 suggest the following points:

1. There is a rise in international inequality between 1960 and 1975 and a decline thereafter up until 1989. This result contrasts with the upward trend observed by Theil and Deepak (1994) since 1975. The discrepancy between the two factual findings seems to be due to the inclusion in our case of China and the USSR together with other countries in the data set. The countries not covered by Theil and Deepak amount to nearly 30% of the world population.
2. For 1989 differences in activity rates account for merely 5% of total income inequality. However, this share has been growing throughout the period. It has to be noted that we have not been able to separate productivity per worker from the employment rate.
3. Since 1975, while the inequality in productivities has been declining, the inequality in activity rates has been rising steadily, both in absolute terms.

In order to carry out a complete analysis with the full factor decomposition, we shall restrict ourselves to the 23 OECD countries for the period 1970–1990. In this case, the previous data set will be supplemented with the information in the *Labour Force Statistics* (OECD). This will allow us to perform the complete decomposition of $T(x)$ into the four factors in (3).

Table 1
Two-factor decomposition of international income inequalities, 1960–89

	$T(x)$	T(yve)	T(avw)
1960	0.5175	0.5009 (96.8%)	0.0166 (3.2%)
1965	0.5766	0.5590 (97.0%)	0.0175 (3.0%)
1970	0.5864	0.5674 (96.8)	0.0190 (3.2%)
1975	0.5924	0.5719 (96.5)	0.0206 (3.5%)
1980	0.5731	0.5501 (96.0)	0.0230 (4.0%)
1985	0.5307	0.5042 (95.0)	0.0265 (5.0%)
1989	0.5393	0.5122 (95.0)	0.0271 (5.0%)

Table 2
Four-factor decomposition of income inequalities within OECD, 1970–90

	$L(x)$	$L(y)$	$L(e)$	$L(a)$	$L(w)$
1970	0.0379	0.0333	0.0012	0.0027	0.0008
		(87.7)	(3.1)	(7.1)	(2.1)
1975	0.0291	0.0255	0.0008	0.0024	0.0004
		(87.6)	(2.8)	(8.3)	(1.3)
1980	0.0271	0.0221	0.0013	0.0035	0.0002
		(81.4)	(4.8)	(12.9)	(0.9)
1985	0.0280	0.0202	0.0039	0.0036	0.0003
		(72.2)	(14.0)	(12.8)	(1.0)
1990	0.0193	0.0130	0.0030	0.0030	0.0004
		(67.1)	(15.7)	(15.4)	(1.8)

The results in Table 2 prompt the following comments:

1. Not surprisingly, overall cross-country inequality is substantially less within the 23 OECD countries than worldwide.
2. The joint role of the activity rate, a, and the working-age population rate, w, in explaining overall inequality is higher among OECD countries than worldwide. Here too the combined role of these two factors doubles its share throughout the period. The increased relative weight of these two factors is caused by the combination of the stability of the absolute level of inequality in the a and w factors, accompanied with a significant fall in the total inequality.
3. The decomposition between the role of productivity per worker differences, y, and that of the national variations in employment rates, e, is empirically relevant. Our results show that in 1990 nearly 20% of their joint inequality is attributable to this second factor[4].
4. The part of total inequality explained by differences in productivities per worker has been declining throughout the period and reached two thirds only by 1990. This is consistent with the expected effects of higher international factor mobility.
5. While international differences in the working-age population ratio and an independent factor do not seem to play any significant role, empirical evidence supports the usefulness of the decomposition into the other three factors. No major loss appears to follow from aggregating factors a and w.

4. Conclusions

1. In this paper we have presented a decomposition of the inequality of per capita national incomes

[4]This empirical result is consistent with the evidence obtained by Esteban (1994) on the decomposition of the regional inequality in per capita incomes within the European Union. In 1990 two thirds of the recorded interregional inequality in the EU correspond to regional differences in productivity, the rest is explained by variations in the employment rates, 17%, and by the active to total population rate (aw), 17%.

into the sum of its factor components. Specifically, we express the inequality in incomes as the sum of the inequalities of (i) the productivity per employed worker, (ii) the employment rate, (iii) the active over working-age population rate, and (iv) the working-age over total population rate.

2. Empirical evidence shows that when the analysis is restricted to homogeneous economic regions, in our case the OECD countries, as much as one third of the income inequality is attributable to factors other than productivity differentials. This evidence suggests that convergence in productivities might be faster than in the other factors.

3. Inequalities in unemployment rates appear to play an increasing role, most specially when we restrict to homogeneous economic regions. A possible explanation is that the furthering of economic integration goes with higher local specialization and that this makes local economies more sensitive to sector-specific shocks.

4. The ratio of active to total population explains a nonnegligible share of total inequality in homogeneous economic regions. We find somewhat surprising that even within these restricted regions there still exist such significant differences in activity rates.

Acknowledgements

Appendix 1

(*a*) *List of sample countries in world inequality analysis* (*n* = 120): Algeria, Angola, Benin, Botswana, Burkina Faso, Burundi, Cameroon, Cape Verde, Central African Rep, Chad, Comoros, Congo, Egypt, Gabon, Gambia, Ghana, Guinea, Guinea–Bissau, Ivory Coast, Kenya, Lesotho, Madagascar, Malawi, Mali, Mauritania, Mauritius, Morocco, Mozambique, Namibia, Niger, Nigeria, Reunion, Rwanda, Senegal, Seychelles, Somalia, South Africa, Swailand, Togo, Tunisia, Uganda, Zaire, Zambia, Zimbabwe, Barbados, Canada, Costa Rica, Dominican Rep., El Salvador, Guatemala, Haiti, Honduras, Jamaica, Mexico, Nicaragua, Panama, Puerto Rico, Trinidad and Tobago, United States, Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador, Guyana, Paraguay, Peru, Suriname, Uruguay, Venezuela, Bangladesh, China, Hong Kong, India, Indonesia, Iran, Israel, Japan, Jordan, Korea Rep., Malaysia, Myanmar, Pakistan, Phillipines, Saudi Arabia, Singapore, Sri Lanka, Syria, Taiwan, Thailand, Austria, Belgium, Cyprus, Czechoslovakia, Denmark, Finland, France, West Germany, Greece, Iceland, Ireland, Italy, Luxembourg, Malta, Netherlands, Norway, Portugal, Romania, Spain, Sweden, Switzerland, Turkey, United Kingdom, USSR, Yugoslavia, Australia, Fiji, New Zealand, Papua N. Guinea.

(*b*) *List of sample countries in developed world inequality analysis* (*n* = 23): Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States.

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