ELSEVIER

Journal of Economic Behavior & Organization
Vol. 32 (1997) 287–300

JOURNAL OF
Economic Behavior
& Organization

# Costly transfer institutions and the core in an overlapping generations model

Merwan Engineer[a,*], Joan Esteban[b], József Sákovics[b]

[a] *Department of Economics, University of Victoria, PO Box 3050 MS 8532, Victoria, BC, V8W 3P5, Canada*
[b] *Institut d'Anàlisi Econòmica, CSIC, Campus Universitat Autònoma de Barcelona, 08193 Bellaterra (Barna), Spain*

Received 25 September 1995

**Abstract**

We examine an overlapping generations economy with no pre-existing intergenerational transfers and study the set of transfer institutions in the core of the economy, that is, institutions that would be admissible for the first generation to create and would not be rejected by any future generation. Institution-building is assumed to be costly. We allow the cost of creating a new institution to vary with the size of the transfer it institutes and show that all institutions in the core must cost as much to build as they transfer. In fact, the core set is nonempty if and only if the institution that supports the golden-rule transfer costs as much to create as it transfers. The core set is characterized for various cost functions. We conclude that costs associated with the creation of transfer institutions are essential to make intergenerational transfers socially viable, but they may induce the choice of institutions making suboptimal transfers.

*JEL classification*: C71; D23; E21; H55

*Keywords*: Core; Institutions; Overlapping generations; Transfers

## 1. Introduction

In his seminal overlapping generations paper, Samuelson (1958) writes:

The Golden Rule or Kant's Categorical Imperative (enjoining like people to follow the common pattern that makes each best off) are often not self-enforcing: if all but one

* Corresponding author. Tel.: (250) 721-8536; fax: (250) 721-6214; e-mail: MEnginee@.UVic.CA.


*PII* S0167-2681(96)00906-7

> obey, the one may gain selfish advantage by disobeying – which is where the sheriff comes in: we politically invoke force on ourselves attempting to make an unstable equilibrium a stable one. (p. 480)

Samuelson, though aware of the incentives for intergenerational conflict, invokes the sheriff to enforce intergenerational cooperation. The bulk of the subsequent overlapping generations literature has blindly followed and implicitly invoked the sheriff to support optimal policies. This has prompted Brock (1990) to strongly criticize this literature as suffering from the *nirvana* fallacy. Much earlier the importance of solving the problems of supporting intergenerational trust are clearly expounded in the classical works of David Hume (1758, 1963) and Jeremy Bentham (1776, 1988). They argue that newly born generations might feel no obligation to abide by a pre-existing social contract that they did not implicitly agree upon.

In recent years a fairly extensive literature has arisen to study this problem from both cooperative and non-cooperative approaches.[1] Yet both methods of analysis yield rather negative results. From the cooperative point of view it is fair to say that no natural solution concept – if it exists at all – has emerged for supporting intergenerational transfers. The obvious cooperative solution concept is the core. However, the core is empty in the most intereresting cases. The alternative cooperative solution concepts launched by Chae (1987) and Aliprantis and Burkinshaw (1990) are also not free of problems (Chae and Esteban, 1993). From the non-cooperative approach results are no better. Not to perform intergenerational transfers is the dominant strategy for each generation, unless we are prepared to work with history-dependent strategies with punishments for deviant generations.[2]

It is our view that societies have solved the problem of supporting intergenerational trust by creating institutions. Institutions may take concrete form such as money and social security with their associated bureaucracies involving markets and taxes. More generally North (1991) describes institutions as "...humanly devised constraints that structure political, economic and social interaction ... consisting of both informal constraints (sanctions, taboos, customs, traditions, and codes of conduct), and formal rules (constitutions, laws, property rights)." Political constitutions, for example, have built-in mechanisms that make them difficult to change.

In this paper we study the role of costly institutions in making intergenerational transfers socially viable.[3] We conceive institutions as norms or laws establishing a constant transfer rule. However, institutions can be set up only at a cost that is assumed to

---

[1]See Aliprantis and Burkinshaw (1990), Chae (1987), Esteban (1986), Esteban and Millan (1990) and Hendricks et al. (1980) on the cooperative approach and Hammond (1975), Kandori (1992) and Salant (1991) on the strategic approach. The existence of competitive equilibrium in the general overlapping generations models is proven in Balasko et al. (1980) and Aliprantis et al. (1989).

[2]Engineer and Bernhardt (1992) criticize history-dependent strategies as requiring unreasonable coordination and excessive levels of communication of both histories and strategies.

[3]This role of institutions and social rules has also been explored by Bryant (1981), Engineer and Bernhardt (1992), Esteban and S ákovics (1993)">Kotlikoff et al. (1988) and Veall (1986), among others.

increase with the size of the transfer it is designed to perform.[4] The economy starts with no institution and it is up to the first newly born generation to choose a transfer institution and pay the corresponding building cost. Future generations can refuse the inherited institution if they are prepared to build a new one which will make them strictly better off while not worsening the well being of subsequent generations. We are interested in characterizing the institutions that are chosen by the initial generation and not refused by any future cohort. We call this set the *core* set.

Our model is Samuelson's canonical overlapping generations model with one perishable commodity and two-period lived agents.[5] The cost of building a transfer institution is allowed to depend on transfers in a general fashion. A necessary condition for an institution to be in the core is that it costs at least as much to build as it transfers. Otherwise, the young generation could gain by not accepting the pre-existing transfer institution and building an identical institution which makes future agents no worse off. A relatively high-cost institution is necessary for stability.

We are able to completely characterize the set of core institutions. A necessary and sufficient condition for non-emptiness of the core set is that the cost of building the *golden-rule institution* is no less than the size of the *golden-rule transfer*. With convex cost functions (decreasing returns to scale) we demonstrate that the core set is a closed interval which contains the golden-rule transfer, but no other Pareto-optimal institution. The core allocations however satisfy the property of *constrained* efficiency. As for concave cost functions, we show that the properties of the core set crucially depend on the specific form of the cost function. We give examples of cost functions for which the core set belongs to the Pareto set as well as examples in which no core institution, but the golden rule, is Pareto efficient.

In a recent paper Esteban and Sákovics (1993) examine a similar model but concentrate on the issue of altering existing institutions at a fixed cost. Unlike them, we do not restrict the generation that builds a new institution to transfer any resources to the existing old generation. Transfers are only mandated for future generations which accept the institution. Thus, the existing old have no implicit power in determining the new institutional structure that will persist into the future. This assumption is consistent with the earlier work on the core (Hendricks et al., 1980 and Esteban, 1986) where the leading generation does not have to start by performing the new transfer in youth. It is also in the spirit of what Bryant (1981) terms the seigniorage problem where the young generation can avoid all obligations to the existing old generation by creating a new money. In the

[4]A key characteristic of institutions is that they are costly to create. Kotlikoff et. al. argue that the bargaining costs incurred by members of a given generation in agreeing to the form of the institution may be substantial. Engineer and Bernhardt (1992) model the cost of a gold-based monetary system not only in terms of the requisite gold but also the costs of setting up the market in which the exchange takes place. They describe the costs of a social security institution as including the costs of designing and agreeing to the form of government as well as the expenditures necessary to set up the bureaucratic apparatus to tax the young and give the proceeds to the old.

[5]As shown in Esteban and Millan (1990), the set of core allocations of this model has essentially the same characteristics as in the model with many commodities and many types of agents when the number of agents per generation becomes arbitrarily large. In other words, in both cases the core set consists of consumption allocations in which no individual agent transfers purchasing power across periods.

conclusion we describe how our model can be generalized to include the Esteban and Sákovics model as a special case.

The paper proceeds to detail the model in Section 2. Section 3 provides a general characterization of the core set whereas Section 4 explores specific cost functions to generate more specific results. Section 5 concludes by discussing the role of costly institutions in supporting intergenerational trust by relating our results to cooperative and strategic models.

## 2. The model

Time is discrete, begins in period 0 and continues indefinitely, $t = 0, 1, 2, \ldots$. At the beginning of each period an agent (representing his generation) is born, indexed by the period of his birth, $t$. All agents are two-period lived and endowments and preferences are identical across generations. Agents in youth are endowed with a homogenous perishable commodity in quantity $\omega_1 > 0$, and in old age agents are endowed with $\omega_2 \geq 0$ of the same commodity. Individual preferences are represented by $u : \mathscr{R}^2_{+,0} \rightarrow \mathscr{R}$. The utility function is assumed to be twice differentiable and to satisfy non-satiation with respect to consumption in youth, $u_1 > 0$, and old age $u_2 > 0$, as well as $u_{12} > 0, u_{11} < 0$ and $u_{22} < 0$.

We shall use the *golden-rule transfer* $x^*$ as a reference point. This transfer is defined by the property that $u(\omega_1 - x^*, \omega_2 + x^*) \geq u(\omega_1 - x, \omega_2 + x)$ for all $x \leq \omega_1$. We assume $u_1(\omega_1, \omega_2) < u_2(\omega_1, \omega_2)$ so that $x^* > 0$; that is, the autarky allocation ($\omega_1$, $\omega_2$) is not Pareto optimal and a positive transfer from young to old is preferred.

As Samuelson points out, social rules and contrivances (like costless to set up markets and costless to produce money) fail to support intergenerational transfers in this environment without the auspices of a sheriff. We alter the environment to consider costly transfer institutions as a mechanism for reallocating resources from the young to the old.

A transfer institution is characterized by its creation cost, $c \geq 0$, and the stationary transfer it implements, $x \geq 0$. As the institution is created for receiving transfers in old age, the costs of its construction are borne by the young generation that creates it. The institution can implement transfer $x$ in all periods following its construction (provided it is not blocked) and can keep on functioning without any additional operating cost.

We allow the cost of creating the institution to depend on the size of the transfer, $c = c(x)$, where $c(0) = 0$. The restriction corresponds to the natural assumption that building no institution has zero direct costs. An institution is *feasible* if $\max\{x, c(x)\} \leq \omega_1$. The set of all feasible institutions is denoted by $X$. We assume that $c(x^*) \leq \omega_1$ so that the golden-rule transfer is always feasible, $x^* \in X$.

We are interested in finding transfer institutions that are in the core of the economy. As the economy starts with no transfer institution, we first start by examining agent 0's decision to create a transfer institution. Agent 0 considers an institution yielding a given transfer $y$ and rejects it in favor of another institution which yields, say, $x$ if (1) the alternative gives future generations at least as much utility as $y$ (and therefore is not more likely to be rejected in the future than $y$) and (2) the alternative institution makes him strictly better off.

Future generations are born in an environment which already contains some transfer institution. They can simply accept the existing institution and perform the corresponding transfer, or reject the status quo, refuse to make any transfer and build a new transfer institution bearing the creation costs. For agents $t \geq 1$, an existing transfer institution $y$ will be rejected if there exists an $x$ giving as much utility to future generations such that creating the new institution makes that agent strictly better off than accepting the status quo.[6] The set of the transfer institutions that will never be rejected is the core of this economy. More formally:[7]

*Definition. A coalition S(r) is the sequence of agents $t \geq r$. A transfer institution y is blocked by coalition S(0) with the transfer institution x if*

$$u(\omega_1 - x, \omega_2 + x) \geq u(\omega_1 - y, \omega_2 + y) \text{ and } u(\omega_1 - c(x), \omega_2 + x) > u(\omega_1 - c(y), \omega_2 + y).$$

A transfer institution $y$ is *blocked* by coalition $S(r), r \geq 1$, with the transfer institution $x$ if

$$u(\omega_1 - x, \omega_2 + x) \geq u(\omega_1 - y, \omega_2 + y)$$

and

$$u(\omega_1 - c(x), \omega_2 + x) > u(\omega_1 - y, \omega_2 + y).$$

A transfer institution belongs to the *core* of the economy, denoted by $C$, if it is not blocked by any coalition $S(r), r \geq 0$.

Finally, we define the sets $R(x)$ and $V(x)$ as,

$$R(x) = \{y \mid u(\omega_1 - y, \omega_2 + y) \geq u(\omega_1 - x, \omega_2 + x)\}, \text{ and}$$
$$V(x) = \{y \mid u(\omega_1 - c(y), \omega_2 + y) > u(\omega_1 - c(x), \omega_2 + x)\}.$$

## 3. The core of the economy

Our first theorem provides a full characterization of the transfer institutions in the core of the economy. Such institutions must (i) cost as much as they transfer and (ii) it must not be possible for a future generation to profitably build another institution that makes subsequent generations no worse off.

*Theorem* 1. *$x \in C$ if and only if: (i) $c(x) \geq x$; and (ii) $R(x) \cap V(x) = \emptyset$.*

Note that (i) is a point condition, it does not impose any global restriction on $c(x)$.

---

[6] Observe that our study also covers the case in which the economy starts with a given transfer institution. Then the analysis simply involves checking whether an institution given at $t = 0$ will be rejected in later periods.

Similarly, the analysis extends to a presence of a pre-existing institution at $t = 0$ which mandates a transfer to a pre-existing old generation at $t = 0$. In this case, agent $t = 0$ is in the same situation as agents $t \geq 1$ in the above analysis.

[7] In order to avoid vacuous generality the definitions below implicitly assume that resources are fully utilized in every period and, therefore, the resulting consumption allocations are weakly Pareto efficient, as defined in Balasko and Shell (1980). This implies that blocking coalitions cannot arise with a finite number of agents.

*Proof.* The stationarity of individual preferences and endowments reduces the types of coalitions to $S(0)$ and $S(1)$. Indeed, all $S(r), r > 0$, will behave identically like $S(1)$.

*Necessity.* Assume first that (i) is not satisfied; i.e. $c(x) < x$. Then, the transfer of $x$ will be blocked by $S(1)$ by creating the transfer institution $x$ anew. Assume now that (ii) is violated so that $\exists y, y \in R(x) \cap V(x)$. Then $x$ can be blocked by $S(0)$ with $y$.

*Sufficiency.* It is clear that if $x$ satisfies (ii) then it cannot be blocked by $S(0)$. Thus, we need only to show that it cannot be blocked by $S(1)$. The fact that (ii) is satisfied implies that for all $y \in R(x)$ we have that $u(\omega_1 - c(y), \omega_2 + y) \leq u(\omega_1 - c(x), \omega_2 + x)$. Inequality (i) then implies that for all $y \in R(x), u(\omega_1 - c(y), \omega_2 + y) \leq u(\omega_1 - x, \omega_2 + x)$. Therefore, no transfer $y$ can be used by coalition $S(1)$ as an alternative to institution $x$. Hence $x \in C$. □

A straightforward property of core allocations is that they are individually rational. In the following remark we show that conditions (i) and (ii) in Theorem 1 imply individual rationality.

*Remark* 1. *If $x \in C$ then $x \in R(0)$.*

*Proof.* Suppose that contrary to our assertion $\exists y \in C$ such that $y \notin R(0)$, i.e.

$$u(\omega_1 - y, \omega_2 + y) < u(\omega_1, \omega_2). \tag{1}$$

Since $y$ satisfies (i) we have

$$u(\omega_1 - c(y), \omega_2 + y) < u(\omega_1, \omega_2). \tag{2}$$

Now, inequality Eq. (1) implies that $0 \in R(y)$, while inequality Eq. (2), taking into account that $c(0) = 0$, implies that $0 \in V(y)$. But the fact that $0 \in R(y) \cap V(y)$ contradicts (ii), and hence the hypothesis that $y \in C$. □

Before turning to our existence theorem we prove a useful lemma.

*Lemma* 1. *Let $y$ be such that $u(\omega_1 - y, \omega_2 + y) < u(\omega_1 - c(x^*), x^*)$. Then $y \notin C$.*

*Proof.* Since $x^* \in R(y)$ by definition, the condition in the Lemma implies that $y$ is blocked by $S(1)$ with $x^*$. □

The following existence theorem proves that the core set is nonempty if and only if the cost of building the golden-rule institution is no less than the size of the golden-rule transfer.

*Theorem* 2. *$c(x^*) \geq x^*$ is necessary and sufficient condition for $C \neq \emptyset$. Moreover, if $C \neq \emptyset$ then $x^* \in C$.*

*Proof. Necessity.* Assume that $c(x^*) < x^*$. Then, for all feasible $y$ we have

$$u(\omega_1 - y, \omega_2 + y) \leq u(\omega_1 - x^*, \omega_2 + x^*) < u(\omega_1 - c(x^*), x^*).$$

Then, by Lemma 1 no feasible $y$ can belong to $C$ and so the core is empty.

*Sufficiency.* Suppose now that the condition $c(x^*) \geq x^*$ is satisfied. We know that the set $R(x^*)$ is a singleton containing $x^*$ only. Further, by the definition of the set $V, x \in V(x)$ for all $x$ and in particular for $x^*$. Therefore $R(x^*) \cap V(x^*) = \emptyset$ and hence $x^* \in C$. The core is nonempty. □

*Corollary* 1. *If $c(x^*) = x^*$ then $C$ is a singleton, containing $x^*$ only.*

*Proof.* If $c(x^*) = x^*$ then by Theorem 2 $x^* \in C$. Let us consider any transfer $y \neq x^*$. Then

$$u(\omega_1 - y, \omega_2 + y) < u(\omega_1 - x^*, \omega_2 + x^*) = u(\omega_1 - c(x^*), \omega_2 + x^*).$$

So $y$ is blocked by $S(1)$ with $x^*$ and the transfer institution $y$ cannot belong to the core. □

## 4. Properties of the core

By Theorem 2 and Corollary 1 the only interesting case to study further is when $c(x^*) > x^*$ (otherwise the core is either empty or it only contains $x^*$). Therefore, in what follows we will assume that this condition holds. The topological structure and efficiency properties of the core depend critically on the form of the cost function.

### 4.1. Decreasing returns to scale

*Assumption* 1. The cost function is strictly increasing, differentiable and convex.

Under this assumption we are able to provide a general characterization of the efficiency of institutions in the core. We use two notions of efficiency in our characterization.

The standard approach is to characterize Pareto-optimal transfers ignoring institutional features. Let $P$ be the set of transfers resulting in Pareto-optimal allocations. It is well known that in the overlapping generations model that $x \in P$ if and only if $x \geq x^*$. Relative to the golden-rule transfer $x^*$, transfers $x > x^*$ make the initial generation better off at the expense of subsequent generations.

An alternative approach is to characterize the Pareto-optimal allocations where the allocations are constrained by the institution technology. Let $CP$ be the set of transfers resulting in constrained Pareto-optimal allocations. It is immediate that under Assumption 1, $x \in CP$ if and only if $x \geq \hat{x}$, where $\hat{x}$ satisfies

$$u(\omega_1 - c(\hat{x}), \omega_2 + \hat{x}) \geq u(\omega_1 - c(x), \omega_2 + x) \text{ for all } x \in X.$$

We term $\hat{x}$ the *constrained golden-rule transfer* because it is the transfer that makes the generation that builds the institution best off. (Note that $\hat{x} = 0$ for all economies such that $c'(0)u_1(\omega_1, \omega_2) \geq u_2(\omega_1, \omega_2)$.)

We now develop some useful results for proving the theorems.

*Lemma* 2. *Let $c$ satisfy Assumption* 1. *Then $V(x)$ is a convex set.*

*Proof.* Let $y, z \in V(x)$, i.e. $u(\omega_1 - c(y), \omega_2 + y) > u(\omega_1 - c(x), \omega_2 + x)$ and $u(\omega_1 - c(z), \omega_2 + z) > u(\omega_1 - c(x), \omega_2 + x)$. By the concavity of $u$, it is clear that for $0 \leq \lambda \leq 1$,

$$u(\omega_1 - [\lambda c(y) + (1-\lambda)c(z)], \omega_2 + \lambda y + (1-\lambda)z) > u(\omega_1 - c(x), \omega_2 + x).$$

By the convexity of $c$ we have that $\lambda c(y) + (1-\lambda)c(z) \geq c(\lambda y + (1-\lambda)z)$. Therefore,

$$u(\omega_1 - c(\lambda y + (1-\lambda)z), \omega_2 + \lambda y + (1-\lambda)z) > u(\omega_1 - c(x), \omega_2 + x)$$

and consequently, $\lambda y + (1-\lambda)z \in V(x)$, for $0 \leq \lambda \leq 1$. Hence $V(x)$ is a convex set. □

Denote by $x'$ the transfer satisfying $c(x) \geq x$ for all $x \geq x'$. Note that because of the convexity of $c$ and the assumption that $c(x^*) > x^*$ it must be that $0 \leq x' < x^*$.

*Lemma* 3. *Let c satisfy Assumption* 1. *Then,* $c(x) - x \geq c(x') - x'$ *for all* $x \geq x'$.

*Proof.* Let $h(x) \equiv c(x) - x$. By the assumption that $c(0) = 0$ and the convexity of $c$, we have that $h(0) = 0$ and that $h$ is also a convex function. Assume now that Lemma 3 is false and that for some $y > x' h(y) < h(x')$. By the convexity of $h$ we can write that $h(\lambda y + (1-\lambda)0) = h(\lambda y) \leq \lambda h(y) + (1-\lambda)h(0) = \lambda h(y)$ for $0 \leq \lambda \leq 1$. In particular, this inequality holds for $\lambda' = x'/y < 1$. That is, $h(x') \leq \lambda' h(y) < h(y)$ and we reach a contradiction. □

The following theorem examines the efficiency of institutions in the core by comparing the core of the economy $C$ to the efficient set $P$. The golden-rule transfer is the lone institution in common to both sets.

*Theorem* 3. *Under Assumption* 1, $C \cap P = x^*$.

*Proof.* Recall that we have assumed that $c(x^*) > x^*$. By Lemma 3 we have that $c(x) > x$ for all $x \in P$ and hence these transfers satisfy condition (i) in Theorem 1. We shall now show that for $x > x^*$ they violate condition (ii).

Let $A(x) = u(\omega_1 - c(x^*), \omega_2 + x^*) - u(\omega_1 - c(x), \omega_2 + x)$ for $x > x^*$. By the concavity of $u$

$$A(x) > (c(x) - c(x^*))u_1(\omega_1 - c(x^*), \omega_2 + x^*) - (x - x^*)u_2(\omega_1 - c(x^*), \omega_2 + x^*).$$

Because of the concavity of $u$ and taking into account that $c(x^*) > x^*$, we have that

$$u_1(\omega_1 - c(x^*), \omega_2 + x^*) > u_1(\omega_1 - x^*, \omega_2 + x^*) \tag{3}$$

$$\text{and } u_2(\omega_1 - c(x^*), \omega_2 + x^*) < u_2(\omega_1 - x^*, \omega_2 + x^*).$$

Substituting, we obtain that

$$A(x) > (c(x) - c(x^*))u_1(\omega_1 - x^*, \omega_2 + x^*) - (x - x^*)u_2(\omega_1 - x^*, \omega_2 + x^*).$$

Since $x^*$ is the golden-rule transfer, $u_1(\omega_1 - x^*, \omega_2 + x^*) = u_2(\omega_1 - x^*, \omega_2 + x^*)$. Hence, by Lemma 3 $A(x) > 0$ for all $x > x^*$, that is, $x^* \in V(x)$. Since $x^* \in R(x)$, it is immediate that $x^* \in R(x) \cap V(x)$ and therefore no Pareto efficient $x \neq x^*$ can be in the core of the economy. □

In our next theorem we prove that $C$ is a convex set.

*Theorem* 4. $x \in C$ if and only if $x \in [\max(\hat{x}, x'), x^*]$.

*Proof.* We first show that if $c(x^*) > x^*$ then $\hat{x} \leq x^*$. We shall do it by contradiction. Assume $\hat{x} > x^*$. We know that by the definition of $V, \hat{x} \notin V(\hat{x})$ and hence by the definition of $\hat{x}$ we deduce that $V(\hat{x}) = \emptyset$. From this it follows that condition (ii) of Theorem 1 is satisfied. By Lemma 3 and the assumptions that $c(x^*) > x^*$ and $\hat{x} > x^*$ we have that condition (i) of Theorem 1 is satisfied also and hence $\hat{x} \in C$. But this contradicts the fact proven in Theorem 1 that if $x \in C$, then $x \leq x^*$. Therefore, it must be that $\hat{x} \leq x^*$.

Note that if $\hat{x} = x^*$, then $C$ is a singleton. Indeed by the definition of $V$ and $R$, when $\hat{x} = x^*$ we have that $x^* \in V(x)$ and $x^* \in R(x)$ for all $x \neq x^*$. Therefore $x^* \in R(x) \cap V(x)$ for all $x \neq x^*$. Hence the core is a singleton and the Theorem holds true.

Let us now examine the case in which $\hat{x} < x^*$. We start by pointing out that because of the convexity of the set $R$ and of the fact that $x^* \in R(x)$ for all $x$ in $X$, we have the following inequalities: if $x \leq x^*$ then for all $y \in R(x), y \geq x$; and if $x \geq x^*$ then for all $y \in R(x), y \leq x$.

We now show the necessity part of the Theorem. Suppose that $\max(\hat{x}, x') = x'$. It is then immediate from Lemma 2 that property (i) in Theorem 1 would not be met by any $x < x'$. Let us suppose then that $\max(\hat{x}, x') = \hat{x}$ and consider $x < \hat{x}$. It is clear that $\hat{x} \in R(x) \cap V(x)$ and hence $x \notin C$.

For sufficiency, we need to demonstrate first that for all $x > \hat{x}$, all $y \in V(x)$ satisfy $y < x$. Indeed, we know that $\hat{x} \in V(x)$ for all $x \neq \hat{x}$. Further, suppose that contrary to our assertion there exists $z > x$ such that $z \in V(x)$. Since $x \in (\hat{x}, z)$ and $x \notin V(x)$ we would contradict the convexity of $V$, proven in Lemma 2. Hence it is true that for all $x > \hat{x}$ all transfers $y \in V(x)$ satisfy $y < x$. Consequently for all $x \in (\hat{x}, x^*], R(x) \cap V(x) = \emptyset$. Since we have already shown that $R(\hat{x}) \cap V(\hat{x}) = \emptyset$, it follows that transfer $x$ satisfies condition (ii) of Theorem 1 if and only if $x \in [\hat{x}, x^*]$. Finally, by Lemma 2 and bearing in mind that $c(x^*) > x^*$, we find that condition (i) of Theorem 1 is satisfied if and only if $x \in [\max(\hat{x}, x'), x^*]$. □

Theorems 3 and 4 together imply that the set $C$ is not a subset of $P$ whenever $C$ contains more than one element. Thus, we find there are economies with non-Pareto optimal transfers in the core. This seemly puzzling result holds because institutions are costly to build. In this case, when evaluating efficiency, Pareto optimality of the transfers is not the appropriate concept to use since it ignores the costs incurred. When the cost of building institutions is included in the efficiency criteria, we find that all core economies are efficient in the sense of being constrained Pareto optimal.

*Corollary* 2. *Under Assumption* 1, $C$ *is a subset of* $CP$.

*Proof.* It follows from Theorem 4, and the fact that $x \in CP$ if and only if $x \geq \hat{x}$. □

### 4.2. Increasing returns to scale

*Assumption* 2. The cost function is strictly increasing, differentiable and strictly concave. Further, $c(0) = 0$.

It is easy to see that under Assumption 2 the set $V(x)$ will not necessarily be convex and that the intersection of the sets $V(x)$ and $R(x)$ need not be convex. Therefore, under increasing returns to scale we cannot expect results as general as found under decreasing returns to scale. We shall now turn to proving three main points:

1. there are economies for which *all* transfers $x$ that satisfy individual rationality, Pareto optimality and $c(x) \geq x$ belong to the core of the economy;
2. there are robust examples of economies which satisfy the conditions in point (1) and that yet do not belong to the core; and
3. there are economies in which non-Pareto optimal transfers are in the core.

Therefore, (1) establishes that increasing returns to scale can produce results that are virtually opposite to decreasing returns to scale (where $x^*$ is the only Pareto optimal institution in the core). Points (2) and (3) show that the conditions individual rationality, Pareto optimality and $c(x) \geq x$ are neither sufficient nor necessary for a transfer institution to be in the core.

*Theorem 5. There exists a cost function c satisfying Assumption* 2 *such that x belongs to the core if and only if* $c(x) \geq x$ *and x is Pareto optimal.*

*Proof.* Recall that $x$ is Pareto optimal if and only if $x \geq x^*$. Differentiating $u(\omega_1 - c(x), \omega_2 + x)$ with respect to the transfer $x$ we obtain,

$$\partial u(\omega_1 - c(x), \omega_2 + x)/\partial x \equiv f(x) = u_2(\omega_1 - c(x), \omega_2 + x) - c'(x)u_1(\omega_1 - c(x), \omega_2 + x)$$

Therefore,

$$f(x^*) = u_2(\omega_1 - c(x^*), \omega_2 + x^*)\left\{1 - \frac{u_1(\omega_1 - c(x^*), \omega_2 + x^*)}{u_2(\omega_1 - c(x^*), \omega_2 + x^*)}c'(x^*)\right\} \tag{4}$$

By Eq. (3) when $c(x^*) > x^*$ we shall have that

$$\frac{u_1(\omega_1 - c(x^*), \omega_2 + x^*)}{u_2(\omega_1 - c(x^*), \omega_2 + x^*)} > \frac{u_1(\omega_1 - x^*, \omega_2 + x^*)}{u_2(\omega_1 - x^*, \omega_2 + x^*)} \tag{5}$$

Consider now cost functions such that $c(x^*) > x^*$, but that $c(x^*) - x^*$ is arbitrarily small. If in addition $c'(x^*) < 1 - \varepsilon$ and $c(x)$ is continuous in a neighborhood of $x^*$, then by continuity of $u_1$, $u_2$ and $c$ we have that for those cost functions $f(x^*) > 0$. By Theorem 1, the values of $x$ that are candidate members of the core have to satisfy $u(\omega_1 - y, \omega_2 + y) \geq u(\omega_1 - c(x^*), \omega_2 + x^*)$. Since $c(x^*)$ is arbitrarily close to $x^*$, the values of $y$ satisfying the above inequality are in the neighborhood of $x^*$ and this neighborhood can be made small enough so that $f(y) > 0$ for all $y$ in this set. Therefore, no $y < x^*$ can belong to the core because by increasing the transfer both the present and future generations would be made better off. Now by the same arguments, for all $x > x^*$ satisfying $c(x) \geq x$, all institutions $y$ that would improve future generations will not make agent $t = r$ better off and therefore $x$ will not be blocked. □

We shall now see that this property which holds for this subset of cost functions does not extend to the full set of cost functions satisfying Assumption 2. Let us assume there exists an $x' > 0$ such that. $c(x') = x'$. Then there exists an $x'' > 0$ such that $c'(x'') = 1$ and $0 < x'' < x'$. Observe that the value of $x''$ is determined by the shape of $c$, whilst the value of $x^*$ depends on the shape of $u$ only. Therefore, we could have either case: $x^* \leq x''$ or $x'' < x^*$. If $x < x^*$, all utility improving transfer institutions, $y \in R(x)$, satisfy that $y \geq x$ and there is a $y^o$ preferred to $x$ such that $x < x^* < y^o$. If $x > x^*$, then the utility improving transfers $y$ satisfy that $x \geq y$ and that there exists a $y'$ such that $x > x^* > y'$.

*Theorem 6.* (i) *There are admissible cost functions such that there are $x \notin P$ that belong to the core; and (ii) there are admissible cost functions for which there are Pareto optimal transfers with $c(x) \geq x$ that do not belong to the core.*

*Proof.* Let us examine once more $f(x^*)$, which is given by Eq. (4). By Eq. (5) we have that $f(x^*) < u_2(\omega_1 - c(x^*), \omega_2 + x^*)\{1 - c'(x^*)\}$. Then, assuming that $c$ is such that $x^* < x''$, we have that $c'(x^*) > 1$ and hence $f(x^*) < 0$. By continuity, $f(x) < 0$ for all $x$ in the neighborhood of $x^*$. Hence, there are transfers $x \in P$, such that $x^* + \varepsilon \geq x > x^*$, that do not belong to the core because they are dominated by $x^*$. By the same argument for $y, x^* > y \geq x^* - \varepsilon$ we have that all $x$ that are utility improving to future generations with respect to $y$ will make agent $t = r$ worse off and thus $y$ will not be blocked. □

The result that there are Pareto-optimal transfer institutions that may not belong to the core does not critically depend on the assumption that $x^* < x''$. We show that in the following theorem.

*Theorem 7. There exist cost functions satisfying Assumption 2 with $x^* > x''$ for which there are transfer institutions $y \in P$, such that $c(y) \geq y$, and which do not belong to the core.*

*Proof.* Let $y'$ satisfy $c(y') = y' > 0$. We shall prove that there are admissible cost functions for which $y'$ does not belong to the core. By continuity this can be extended to other values of x close enough to $y'$. Let us evaluate $f(y')$. We have that

$$f(y') = u_2(\omega_1 - c(y'), \omega_2 + y')\left\{1 - c'(y')\frac{u_1(\omega_1 - c(y'), \omega_2 + y')}{u_2(\omega_1 - c(y'), \omega_2 + y')}\right\}$$

$$= u_2(\omega_1 - y'), \omega_2 + y')\left\{1 - c'(y')\frac{u_1(\omega_1 - y', \omega_2 + y')}{u_2(\omega_1 - y', \omega_2 + y')}\right\}.$$

Since $y' > x^*$, we know that $u_1(\omega_1 - y', \omega_2 + y')/u_2(\omega_1 - y', \omega_2 + y') > 1$. We shall denote by $m$ this marginal rate of substitution. Further, we know that $c'y' < 1$. Therefore, we cannot establish the sign of $f(y')$.

Let us now consider the family of cost functions $c_\lambda$ defined as

$$c_\lambda(x) = \lambda c(x) + (1 - \lambda)x, \; 0 < \lambda \leq 1.$$

It is immediate that for $c(x)$ admissible, all $c_\lambda$ are admissible cost functions. In addition, they have the following properties: (i) $y'_\lambda = y'$, (ii) $x''_\lambda = x''$, (iii) $c'_\lambda(y') = \lambda c'(y') + (1 - \lambda)$. By property (i) it is clear that the marginal rate of substitution at $y'_\lambda$ is equal to $m$ for all $\lambda$. By property (ii) we have that $x''_\lambda < x^*$ for all $\lambda$. Finally, by property (iii) there exists $\lambda_o > 0$ such that $c'_{\lambda_o}(y')m > 1$. This is the case for all $\lambda, 1 > \lambda > (m-1)/m(1-c'(y'))$. For such $\lambda$ we have that $f(y', \lambda) < 0$. Hence, $y'$ is dominated by $y' - \varepsilon$ which makes generation $t = r$ as well as all future generations better-off. □

## 5. Conclusion

In this paper we examine an overlapping generations economy with no pre-existing intergenerational transfers and study the institutions that would be admissible for the first generation to create and would not be rejected by any future generation. The key feature needed for institutions to support Pareto-improving transfers is that they be costly to create. In fact, a necessary condition for institutions to be in the core is that they cost as much to build as they transfer. Otherwise, a young generation could benefit by not performing by the existing institution and building an identical institution for old age. A much stronger result is that the core set is non-empty if and only if the golden-rule institution costs as much as the golden-rule transfer, $c(x^*) \geq x^*$. It is worth stressing that the non-emptiness of the core depends on a property of the cost function that has to be satisfied only at one point.

The shape of the cost function on its support determines the shape of the core set. With convex cost functions we are able to completely characterize the core. When $c(x^*) = x^*$, the core set only contains singleton $x^*$. On the other hand when $c(x^*) > x^*$, the core set is the closed interval $[\hat{x}, x^*]$, where $x^*$ is the only Pareto-efficient transfer in the interval. Observe now, that if the first generation could freely choose its most preferred institution from the core set, it would choose $\hat{x}$ rather than $x^*$. Therefore, the existence of costs associated with the creation of transfer institutions is essential to make intergenerational transfers socially viable, but it may induce the choice of institutions making Pareto-inefficient transfers.[8]

The results of Esteban and Sákovics (1993) provide an interesting comparison with our analysis. Like us, they find that the golden-rule institution is in the core. Unlike us, they find cooperation can be supported with small costs and the core is never empty. In fact, as the cost becomes smaller, transfers in the core converge to the golden-rule transfer.

What can explain the different results? In Esteban and Sákovics (1993) a deviating young generation can change the level of transfer at a fixed cost but is restricted to making the new transfer to the existing old generation. This specification can be translated into our model by assuming that the fixed cost and the transfer enter the cost

[8]In a similar model where generations play strategically against each other and are limited to history-independent strategies, Engineer and Bernhardt (1992) find that the building generation chooses the constrained golden-rule institution. This suggests that both strategic and cooperative approaches yield similar implications for the choice of institutions.

We are grateful to a referee for pointing out that costly institutions add some determinancy to the overlapping generations model. Versions of the model with a price system and sheriff typically yield a wide range of equilibria most of which are not Pareto optimal (see Gale, 1973).

function linearly; that is, $c(x) = \alpha + x$, where $\alpha > 0$ is the fixed cost.[9] Note that this cost function implies $c(x^*) > x^*$ so that it always satisfies the necessary and sufficient condition for $x^*$ belonging to the core, consistent with Esteban and Sákovics.

The present model generalizes the one developed in Esteban and Sákovics (1993) in two respects. On the one hand, as we have just argued, the cost function assumed here is sufficiently general to incorporate the restriction that the deviating agents have to perform by the new rule together with the fixed cost as assumed there. On the other hand, in the present paper we work with a much broader class of individual preferences.

## Acknowledgements

We would like to thank two anonymous referees for the their detailed and helpful comments.

## References

Aliprantis, C.D., D.J. Brown and O. Burkinshaw, 1989, Equilibria in exchange economies with a countable number of agents, Journal of Mathematical Analysis and Applications, 142, 250–299.

Aliprantis, C.D. and O. Burkinshaw, 1990, An overlapping generations model core equivalence theorem, Journal of Economic Theory, 50, 362–380.

Balasko, Y., D. Cass and K. Shell, 1980, Existence of competitive equilibrium in a general overlapping-generations model, Journal of Economic Theory, 23, 307–322.

Balasko, Y. and K. Shell, 1980, The overlapping generations model I: The case of pure exchange without money", Journal of Economic Theory, 23, 281-306.

Bentham, J., 1988, A Fragment on Government, (Cambridge University Press, New York).

Brock W.A., 1990, The overlapping generations model with money and transactions costs, In: B.M. Friedman and F. Hahn, Eds., Handbook of Monetary Economics, (Elsevier Science Publishers) pp. 264–295.

Bryant, J., 1981, The competitive provision of fiat money, Journal of Banking and Finance, 5, 587–593.

Chae, S., 1987, Short run core equivalence in an overlapping generations model, Journal of Economic Theory, 43, 170–183.

Chae, S. and J.M. Esteban, 1993, Core equivalence in an overlapping generations model, Journal of Economic Theory, 59, 417–425.

Engineer, M. and D. Bernhardt, 1992, Endogenous transfer institutions in overlapping generations, Journal of Monetary Economics, 29, 445–447.

Esteban, J.M., 1986, A characterization of the core in overlapping generations economies, Journal of Economic Theory, 39, 439–456.

Esteban, J.M. and T. Millan, 1990, Competitive equilibria and the core of overlapping generations economies, Journal of Economic Theory, 50, 155–174.

Esteban, J.M. and Sákovics, J., 1993, Intertemporal transfer institutions, Journal of Economic Theory, 61, 189–205.

Gale, D., 1973, Pure exchange equilibrium of dynamic economic models, Journal of Economic Theory, 6, 12–36.

Hammond, P., 1975, Charity: Altruism or cooperative egoism?, In: E.S. Phelps, Ed., Altrusim, Morality and Economics Theory, (Russel Sage Foundation, New York), pp.115–131.

[9]This generalization requires that the transfer actually be paid to the existing old. This feature could be easily added to our analysis but it would not change the results because the criteria for blocking an institution only involve the current young and future generations.

Hendricks, K., K. Judd and D. Kovenock, 1980, A note on the core in the overlapping generations model, Economic Letters, 6, 95–97.

Hume, D., 1963, Essays Moral, Political and Literary, (Oxford University Press, London).

Kandori, M., 1992, Repeated games with overlapping generations of players, Review of Economic Studies, 59, 81–92.

Kotlikoff, L.J., T. Persson and L.E.O. Svensson, 1988, Social contracts as assets: A possible solution to the time-consistency problem, The American Economic Review, 78, 662–677.

North, D., 1991, Institutions, Journal of Economic Perspectives, 5, 97–112.

Salant, D.J., 1991, A repeated game with finitely lived overlapping generations of players, Games and Economic Behavior, 3, 244–259.

Samuelson, P.A., 1958, An exact consumption-loan model of interest with or without the social contrivance of money, Journal of Political Economy, 66, 467–482.

Veall, M.R., 1986, Public pensions as optimal social contracts, Journal of Public Economics, 31, 237–251.