

Competitive Equilibria and the Core of Overlapping Generations Economies

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This paper analyzes whether the well known result that for Arrow-Debreu economies all competitive equilibria belong to the core holds true in overlapping generations economies. In these economies competitive equilibria are not a subset of the core. We define the classical set as the set of all those competitive allocations which cannot be affected by a monetary tax-transfer policy. Then, we demonstrate that every competitive equilibrium in the classical set belongs to the core of the economy. Further, we show that for large economies no monetary equilibrium belongs to the core. *Journal of Economic Literature* Classification Numbers: 021, 023, 111. © 1990 Academic Press, Inc.

1. INTRODUCTION

Overlapping generations models have recently been analyzed from a game theoretic point of view. Specifically, the core of economies with an overlapping generations structure has been studied by Hendricks *et al.* [15], Kovenock [17], Esteban [9], and Chae [6], showing that competitive equilibria may not belong to the core. Related concepts such as bounded core and short-run core have been dealt with by Hendricks *et al.* [15], Chae [6], Chae and Esteban [7], and Esteban [10]. In this paper

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we demonstrate that the well known result that for Arrow–Debreu economies all competitive equilibria belong to the core may hold true for overlapping generations economies when we restrict to a specific set of consumption allocations, the “classical set.”

For finite horizon Arrow–Debreu economies the relations between efficiency, competitive equilibria, and core have been established in a series of well known theorems, as in Arrow and Hahn [1] for instance. On the one hand, the Fundamental Welfare Theorems state that competitive equilibria are Pareto optimal and that Pareto optimal allocations are implementable as competitive equilibria. On the other hand, we have the result that every consumption allocation implementable as a competitive equilibrium belongs to the core of the economy for all the endowments allocations for which it can be obtained as a competitive equilibrium.

Samuelson [19] himself made it plain that the Fundamental Welfare Theorems are not valid in overlapping generations economies. However, in an important paper Balasko and Shell [3] have demonstrated that an equivalence between competitive and efficient allocations can be obtained by an appropriate redefinition of efficiency. They introduce the notion of weak Pareto optimality and demonstrate that it is satisfied by every competitive equilibrium. Likewise, Chae’s [6] work can be seen as an attempt at saving the relation between competitive equilibria and core by redefining the notion of socially viable allocations. He proposes the concept of the bounded core and shows that the competitive equilibria of the economy belong to the bounded core.

In this paper we examine the relation between the notions of Pareto optimality, competitive equilibria, and core. At variance with the works reported above, our approach does not consist in redefining the concepts of dynamic efficiency and/or core. Instead, we shall establish the relations between the standard concepts but restricted to the “classical set”. The classical set, \mathcal{C} , consists of those competitive equilibrium allocations which cannot be affected by monetary tax-transfers policies, as studied by Balasko and Shell [4, 5]. Specifically our results are the following: (i) the classical set is a subset of the set of Pareto optimal allocations; and (ii) every competitive equilibrium in the classical set belongs to the core. The fact that competitive equilibria not in the classical set may not be Pareto optimal or not belong to the core suggests that the fundamental singularity of overlapping generations models lies in the possibility of competitive equilibria in which consumers violate their budgets in every period.

Let us now briefly discuss the relation between these results and the existing literature. For the case of exchange economies with one agent per generation, Hendricks *et al.* [15] and Esteban [9] have demonstrated that all Pareto optimal Walrasian equilibria belong to the core. We know that in overlapping generations economies Walrasian equilibria might not be

Pareto optimal and thus cannot belong to the core. It can be tempting to conclude, as suggested by Hendricks *et al.* [5], that the only reason why Walrasian equilibria may not belong to the core is that they may fail to be dynamically efficient. That this is not the case for economies with many agents per generation has been made plain by Kovenock [17] through an example of a Pareto optimal Walrasian equilibrium which does not belong to the core, and the core being empty. In this respect, we give here another example of a Pareto optimal Walrasian allocation which does not belong to the core, while the core is not empty, and show that these examples are by no means pathological. We demonstrate in Proposition 3 that for every competitive consumption allocation with a price sequence such that $\text{Lim inf}_{t \rightarrow \infty} \|p^t\| \geq \varepsilon > 0$ one can construct a distribution of initial endowments for which that equilibrium consumption allocation is not in the core. Moreover, we prove in Proposition 2 that $\text{Lim inf}_{t \rightarrow \infty} \|p^t\| = 0$ is a sufficient condition for a competitive equilibrium to belong to the core of the economy, irrespective of the distribution of endowments.¹ This result is stronger than Chae's [6] Theorem 4.1, where with a continuum of agents he finds that a sufficient condition for an allocation to belong to the core is that the present value of the total endowment be finite.

Besides the eventual interest of these results for general equilibrium theory, the propositions presented in this paper have a special bearing on monetary theory. Specifically, monetary and IOU equilibria can never satisfy the sufficiency condition on prices for a competitive equilibrium to belong to the core. Moreover, we show that every monetary equilibrium becomes excluded from the core upon replication of the economy. These results are rather negative concerning the suitability of overlapping generations models for the analysis of fiat money. In our discussion at the end of the paper we suggest that our results can be interpreted as a formal demonstration of Clower's [8] observation that, as quoted in de Vries [21], "money differs from other commodities in being universally acceptable as an exchange intermediary by virtue not of individual choice but rather by virtue of social contrivance" (pp. 14–15).

The paper is structured as follows. Section 2 contains the description of the model and the basic assumptions. In Section 3 we define and characterize the classical set. Section 4 provides an example of a Walrasian Pareto optimal equilibrium which does not belong to the core, while the core is not empty. The relation between efficiency, competitive equilibria, and the core of the economy is the object of Section 5. There we state our main propositions. Section 6 focusses on monetary equilibria and we prove that monetary equilibria do not belong to the core of large economies. The paper ends with a discussion of the implications of our propositions for

¹ The fiber structure of the equilibrium set is crucial for these results; see Balasko [2].

monetary theory. We examine the relation with the work of Douglas Gale [12] on the trustworthiness of intertemporal allocations and the role of money. We argue that our results provide a rationale for the lack of trust in the IOU competitive equilibria.

2. NOTATION, ASSUMPTIONS, AND DEFINITIONS

We shall assume a pure exchange economy with l perishable commodities available at every date. In every period t , $t = 1, 2, \dots$, a number n of agents are born² and live for two periods. At the beginning of this economy there exists a generation previously born at time $t = 0$. Let $x_{t,j}^{t+s,i}$ be the consumption of good i ($i = 1, 2, \dots, l$) at period $t + s$ ($s = 0, 1$) ($t = 1, 2, \dots$) by consumer j ($j = 1, 2, \dots, n$) born at t ($t = 0, 1, 2, \dots$); $x_{t,j}^{t+s} \in \mathbb{R}_+^l$ be the consumption vector at period $t + s$ ($s = 0, 1$) by agent j born at t ; $x_{t,j} \in \mathbb{R}_+^{2l}$ the vector of consumptions corresponding to the two periods that agent will be alive; $x_t^{t+s} \in \mathbb{R}_+^l$ be the vector of consumptions by generation t at period $t + s$; and $x_t \in \mathbb{R}_+^{2l}$ the vector of consumptions of generation t . For generation $t = 0$ we have that $x_0 = x_0^1$ and, of course, $x_0 \in \mathbb{R}_+^l$. We shall use x to denote the sequence $x = \{x_0, x_1, x_2, \dots\}$. Similarly, we shall denote by $\omega_{t,j}^{t+s,i}$ the endowment of good i at $t + s$ by agent j born at period t , by $\omega_{t,j}^{t+s} \in \mathbb{R}_+^l$ the endowment vector at period $t + s$ ($s = 0, 1$) of agent j born at t , by $\omega_{t,j} \in \mathbb{R}_+^{2l}$ the vector of endowments corresponding to the two periods that agent j will be alive, by $\omega_t^{t+s} \in \mathbb{R}_+^l$ the vector of endowments of generation t at period $t + s$, and by $\omega_t \in \mathbb{R}_+^{2l}$ the vector of endowments of generation t . For generation $t = 0$ we have that $\omega_0 = \omega_0^1$ and $\omega_0 \in \mathbb{R}_+^l$. Finally, we shall denote by ω the sequence $\omega = \{\omega_0, \omega_1, \omega_2, \dots\}$. Let Ω be the set of all sequences ω which are uniformly bounded from above.

Let us denote by $\bar{\omega}^t$ the aggregate endowments available at period t and by $\bar{\omega}$ the sequence $\bar{\omega} = \{\bar{\omega}^1, \bar{\omega}^2, \dots\}$. Since $\omega \in \Omega$, it is obvious that $\bar{\omega} \in \Omega$ as well. Given a sequence of aggregate endowments $\bar{\omega}$, we shall denote by $\omega(\bar{\omega})$ the set of sequences of individual initial endowments such that

$$\omega'_{t-1} + \omega'_t = \bar{\omega}_t, \quad \omega'_{t-s} \in \mathbb{R}_+^n, \quad s = 0, 1 \quad \text{and} \quad t = 1, 2, \dots$$

Preferences of consumer j born at t can be represented by a utility function $u_{t,j}: \mathbb{R}_+^{2l} \rightarrow \mathbb{R}$, for $t = 1, 2, \dots$, and $u_{0,j}: \mathbb{R}_+^l \rightarrow \mathbb{R}$, for $t = 0$, with $j = 1, \dots, n$.

Assumption 1. $u_{t,j}$ has strictly positive first order partial derivatives and is strictly quasi-concave.

² The model can be trivially generalized to a variable number of agents per generation $n(t)$ uniformly bounded above by a finite number n .

DEFINITION 1. A consumption allocation sequence x is *feasible* if

$$x'_{t-1} + x'_t = x' \leq \omega'_{t-1} + \omega'_t = \omega' \quad \text{for } t = 1, 2, \dots$$

DEFINITION 2. The feasible consumption allocation sequence x is *Pareto optimal* if there is no feasible \tilde{x} such that $u_{t,j}(\tilde{x}_{t,j}) \geq u_{t,j}(x_{t,j})$ for $t = 0, 1, 2, \dots$ and $j = 1, \dots, n$ with at least one strict inequality.

DEFINITION 3. The feasible consumption allocation sequence x is *weakly Pareto optimal* if there is no feasible \tilde{x} , with $\tilde{x}_{t,j} = x_{t,j}$, except for a finite number of periods, and such that $u_{t,j}(\tilde{x}_{t,j}) \geq u_{t,j}(x_{t,j})$ for $t = 0, 1, 2, \dots$ and $j = 1, \dots, n$, with at least one strict inequality.

Full information and perfect foresight are assumed throughout the paper. Let $p^{t,i}$ denote the price of commodity i delivered at period t , $i = 1, \dots, l$ and $t = 1, 2, \dots$, $p^t \in \mathbb{R}^l_+$ the vector of prices at t , and p the price sequence $p = \{p^1, p^2, \dots\}$. We shall normalize prices by setting $p^{1,1} = 1$ and denote by \mathcal{P} the set of such price sequences, i.e., $\mathcal{P} = \{p/p^{1,1} = 1, p^t \in \mathbb{R}^l_+\}$.

Following Balasko and Shell's [4, 5] we shall now introduce the notion of a price-income equilibrium.

DEFINITION 4. Let x be a consumption allocation sequence. We shall say that the *price sequence* $p \in \mathcal{P}$ *supports* x if we have that $u_{t,j}(x_{t,j}) \geq u_{t,j}(\tilde{x}_{t,j})$ for all $\tilde{x}_{t,j}$ satisfying

$$p^t \cdot x''_{t,j} + p^{t+1} \cdot x^{t+1}_{t,j} \geq p^t \cdot \tilde{x}''_{t,j} + p^{t+1} \cdot \tilde{x}^{t+1}_{t,j} \\ \text{for } j = 1, \dots, n \text{ and } t = 1, 2, \dots,$$

and for $t = 0$, $u_{0,j}(x_{0,j}) \geq u_{0,j}(\tilde{x}_{0,j})$ for all $\tilde{x}_{0,j}$ satisfying

$$p^1 \cdot x^1_{0,j} \geq p^1 \cdot \tilde{x}^1_{0,j}, \quad j = 1, \dots, n.$$

Further, for the sequence $w = \{w_0, w_1, w_2, \dots\}$, $w_t \in \mathbb{R}^n_+$, we shall say that (p, w) is a *price-income equilibrium* if p supports x and if w satisfies $p^t \cdot x''_{t,j} + p^{t+1} \cdot x^{t+1}_{t,j} = w_{t,j}$ for $j = 1, \dots, n$ and $t = 1, 2, \dots$, and $p^1 \cdot x^1_{0,j} = w_{0,j}$ for $j = 1, \dots, n$.

We shall now introduce the notion of competitive equilibrium.

DEFINITION 5. A *competitive equilibrium* is a sequence of commodity prices $p \in \mathcal{P}$, vector $\mu = (\mu_1, \dots, \mu_n)$, $\mu \in \mathbb{R}^n$, $\sum_j |\mu_j| = |\sum_j \mu_j|$, and endowments allocation sequence $\omega \in \Omega$, and a consumption allocation sequence x such that:

- (i) $p^t \cdot x''_{t,j} + p^{t+1} \cdot x'^{t+1}_{t,j} \leq p^t \cdot \omega'_{t,j} + p^{t+1} \cdot \omega^{t+1}_{t,j}$ and $u_{t,j}(x_{t,j}) \geq u_{t,j}(\tilde{x}_{t,j})$ for all \tilde{x}_t satisfying the budget constraint, for $t = 1, 2, \dots$, and $j = 1, \dots, n$, and for $t = 0$ $p^1 \cdot x^1_{0,j} \leq p^1 \cdot \omega^1_{0,j} + \mu_j$ and $u_{0,j}(x_{0,j}) \geq u_{0,j}(\tilde{x}_{0,j})$ for all $\tilde{x}_{0,j}$ satisfying the budget constraint; and
- (ii) $x'_{t-1} + x'_t = \omega'_{t-1} + \omega'_t$ for $t = 1, 2, \dots$

A *monetary equilibrium* is a competitive equilibrium where $\mu > 0$.

Throughout the paper we shall make extensive use of Balasko and Shell's [3] characterization of Pareto optimum allocations.³ Therefore, we shall assume that the conditions given in their Theorem 5.3 are satisfied.

Assumption 2. (a) The Gaussian curvature of consumer (t, j) 's indifference surface through $x_{t,j}$, $0 < x_{t,j} < [x^t, x^{t+1}]$, is uniformly bounded above and below away from zero;

(b) There exists a constant Π , independent of t such that the supporting prices of x satisfy

$$0 < \Pi \leq p^{t,i} / \|p^t\|, \quad \text{for } i = 1, \dots, l \text{ and } t = 1, 2, \dots;$$

(c) The sequence x is uniformly bounded above and below by a strictly positive vector.

We shall borrow from Esteban [9] the definitions of coalition and core of the economy and adapt them to the case of many consumers per generation.

DEFINITION 6. A *coalition* is a non-empty connected subset S of the set of all agents.

We shall denote by S_t the set of all agents born at period t which belong to coalition S , so that $S_t \subseteq S$. Then by the connectedness of S we mean that if $S_r \neq \emptyset$ and $S_{t+k} \neq \emptyset$, then $S_{t+r} \neq \emptyset$, $r = 1, \dots, k-1$. A coalition will thus be formed by a chain of generations, which might not include all their members. We shall denote by f the first of such generations, i.e., $f = \min\{t/S_t \neq \emptyset\}$.

DEFINITION 7. An allocation \bar{x} is *blocked by coalition* S if there exists another allocation x such that:

³ As proven by Millán [18], Balasko and Shell's [3] characterization of Pareto optimal allocations can be extended to economies with many agents.

(i) x is feasible for coalition S , i.e.,

$$\sum_{j \in S_{t-1}} x''_{t-1,j} + \sum_{j \in S_t} x''_{t,j} = \sum_{j \in S_{t-1}} \omega''_{t-1,j} + \sum_{j \in S_t} \omega'_{t,j}, \quad \text{for all } S_t \in S,^4$$

(ii) $u_{t,j}(x_{t,j}) \geq u_{t,j}(\bar{x}_{t,j})$ for all agents $(t, j) \in S$ with at least one strict inequality for some agent $(t, j) \in S_t$.

DEFINITION 8. The *core* of an economy is the set of all allocations that are feasible and not blocked by any coalition.

3. THE CLASSICAL SET: DEFINITION AND CHARACTERIZATION

We shall now formally define and characterize the classical set. In order to do so we shall need the notion of consistency as introduced in Balasko and Shell [4, 5].⁵

DEFINITION 9. We shall say that the endowments allocation sequence $\omega \in \Omega$ is *consistent* with the price-income equilibrium (p, w) associated with the consumption allocation sequence x if

$$p^t \cdot x''_{t,j} + p^{t+1} \cdot x^{t+1}_{t,j} = p^t \cdot \omega'_{t,j} + p^{t+1} \cdot \omega^{t+1}_{t,j} = w_t$$

for $t = 1, 2, \dots$, and $j = 1, \dots, n$.

We shall denote by $\Omega(x)$ the set of sequences ω consistent with the price-income equilibrium associated with x . Let us now introduce the notion of sequential consistency, in which the single condition of life-time income consistency is replaced by a sequence of consistency conditions, one for every period. More precisely, we require this condition to be satisfied in the long run.

Let us define $b'_{t,j} = \max\{p^t \cdot [x^t_{t,j} - \omega^t_{t,j}], 0\}$, $j = 1, \dots, n$ and $t = 1, 2, \dots$; $b'_t = \sum_j b'_{t,j}$; and $B'_t = \max\{b'_{t,1}, \dots, b'_{t,j}, \dots, b'_{t,n}\}$.

DEFINITION 10. We shall say that $\omega \in \Omega$ is *sequentially consistent* with the price-income equilibrium (p, w) associated with x if $\omega \in \Omega(x)$ and $\text{Lim inf}_{t \rightarrow \infty} B'_t = 0$.

⁴ Note that $S_{j-1} = \emptyset$ and hence for the first generation the feasibility condition reads

$$\sum_{j \in S_t} x^t_{t,j} = \sum_{j \in S_t} \omega^t_{t,j}$$

⁵ For the relationship between the notion of consistency with respect to a price income equilibrium and the fiber structure of the equilibrium manifold, see Balasko [2].

We shall denote by $\Omega^s(x)$ the set of endowment allocations ω sequentially consistent with the price-income equilibrium associated with x . Obviously $\Omega^s(x) \subseteq \Omega(x)$. As we shall see, there are consumption allocations such that $\Omega^s(x) = \Omega(x)$, that is, that can only be implemented as equilibria with the property that in the long run every agent balances his budget constraint in every period of his life. Following Balasko and Shell [4, 5], we can interpret this property as implying that the government cannot alter the equilibrium allocation by means of monetary taxes and/or transfers, i.e., there are no bona fide monetary policies. It is in this sense that we borrow from David Gale [11] the word *classical* to denote the set of equilibrium allocations in which there is no room for monetary policies. In other words, allocations in the classical set cannot be competitive equilibria with either inside or outside money.

We are now in a position to define the classical set.

DEFINITION 11. We shall say that the consumption allocation sequence x belongs to the *classical set*, $x \in \mathcal{C}$, if and only if $\Omega^s(x) = \Omega(x)$, i.e., all $\omega \in \Omega(x)$ are sequentially consistent with the corresponding price-income equilibrium (p, w) .

The following proposition provides a characterization of the consumption allocations sequences in the classical set by their supporting prices.

PROPOSITION 1. *Let Assumptions 1 and 2 be satisfied. The consumption allocation sequence x belongs to the classical set, $x \in \mathcal{C}$, if and only if its sequence of supporting prices satisfies that*

$$\liminf_{t \rightarrow \infty} \|p^t\| = 0.$$

Proof. We shall first show that $\liminf_{t \rightarrow \infty} \|p^t\| = 0$ implies that $\liminf_{t \rightarrow \infty} B_t^t = 0$.

Let us denote by the superscript k the agent with the largest amount of borrowing in each generation, i.e., $B_t^t = p^t \cdot [x'_{t,k} - \omega'_{t,k}]$.

It is obvious that

$$\|p^t\| \cdot \|x'_{t,k} - \omega'_{t,k}\| \geq p^t \cdot [x'_{t,k} - \omega'_{t,k}] = B_t^t.$$

By Assumption 2(c) the value of $\|x'_{t,k} - \omega'_{t,k}\|$ is uniformly bounded above by some $K < +\infty$. Thus we can write the inequalities

$$\|p^t\| K \geq \|p^t\| \cdot \|x'_{t,k} - \omega'_{t,k}\| \geq p^t \cdot [x'_{t,k} - \omega'_{t,k}] = B_t^t \geq 0.$$

Therefore when $\liminf_{t \rightarrow \infty} \|p^t\| = 0$ it must be that $\liminf_{t \rightarrow \infty} B_t^t = 0$.

We shall now demonstrate that when $\liminf_{t \rightarrow \infty} \|p^t\| \geq \lambda > 0$ there

exists $\omega \in \Omega(x)$ such that $\text{Lim inf}_{t \rightarrow \infty} B'_t \geq \beta > 0$. For that we can just follow the same steps as in Proposition 4 in Esteban [9]. Consider for instance the endowments allocation sequence ω and the sequence of real numbers $b = \{b_1, b_2, \dots, b_t, \dots\}$ such that $\omega_{t,j} = x_{t,j}$ $j = 3, 4, \dots, n$ and $t = 0, 1, 2, \dots$ and $\omega_{t,1}$ and $\omega_{t,2}$ satisfying

$$\begin{aligned} p^t \cdot [x'_{t,1} - \omega'_{t,1}] &= b_t = -p^t \cdot [x'_{t,2} - \omega'_{t,2}] \\ &= -p^{t+1} \cdot [x'_{t,1} - \omega'_{t,1}] = p^{t+1} \cdot [x'_{t,2} - \omega'_{t,2}]. \end{aligned}$$

Observe that $b_t = B'_t$. By Proposition 4 in Esteban [9] the sequence b exists and is uniformly bounded below by some $\beta > 0$. Further, the endowments allocation sequence thus constructed is such that $\omega \in \Omega(x)$ and hence x is a competitive equilibrium. Q.E.D.

4. COMPETITIVE EQUILIBRIA AND THE CORE: AN EXAMPLE

We shall start by giving an example of an economy in which a Pareto optimal competitive equilibrium without outside, fiat money does not belong to the core of the economy, while the set of core allocations is non-empty.⁶

Consider a one-good economy with two agents a and b per period with identical preferences $u(x'_t, x'^{t+1}) = [x'_t x'^{t+1}]^{1/2}$ for $t = 1, 2, \dots$ and $u(x^1_0) = [x^1_0]^{1/2}$ for agents of generation $t = 0$. Agents have endowments $\omega_{t,a} = (1.5, 0.5)$ and $\omega_{t,b} = (0.5, 1.5)$ for $t = 1, 2, \dots$ and $\omega_{0,a} = 0.5$ and $\omega_{0,b} = 1.5$ for $t = 0$. See Fig. 1.

The consumption allocation $x''_{t,a} = x'_{t,a} = x'_{t,b} = x''_{t,b} = 1$ is a Walrasian equilibrium with equilibrium price sequence $p^t = 1$, for $t = 1, 2, \dots$, and $x^1_{0,a} = 0.5$ and $x^1_{0,b} = 1.5$. This Walrasian allocation is Pareto optimal because the sum of the inverse of prices diverges, thus satisfying Balasko and Shell's criterion. This allocation, however, does not belong to the core because it would be blocked by the coalition formed by agents (t, a) $t = 1, 2, \dots$ with consumption allocation $\tilde{x}_{1,a}(1.5, 1)$ and $\tilde{x}_{t,a} = (1, 1)$ for $t = 2, 3, \dots$. Note that the original consumption allocation is a Walrasian equilibrium with inside money with agents of type "a" behaving as lenders and those of type "b" as borrowers. The blocking coalition is formed by the sequence of lenders who prefer to substitute outside money for inside money.

⁶ Kovenock [17] has produced another example of a Pareto optimum Walrasian equilibrium not in the core, where the set of core allocations is empty. In his example the two agents of any generation have preferences defined on different goods. Only the two members of generation $t = 0$ share their preferences for one common good.

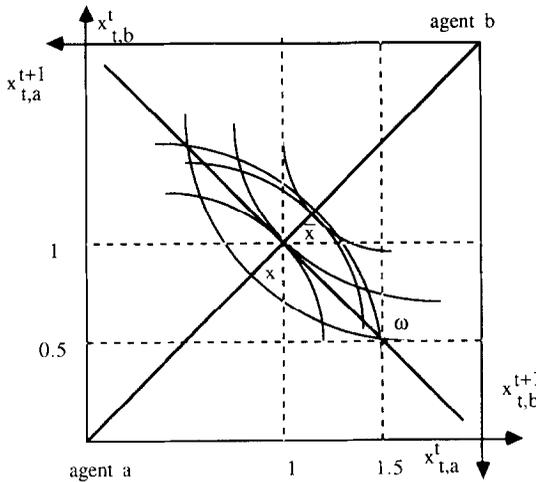


FIGURE 1

The set of core allocations is not empty. Consider for instance the weakly Pareto optimal allocation $\bar{x}_{0,a}^1 = 0.5$ and $\bar{x}_{0,b}^1 = 1.5$, $\bar{x}_{t,a} = (1.1, 1.1)$ and $\bar{x}_{t,b} = (0.9, 0.9)$, $t = 1, 2, \dots$. This allocation has supporting prices $p^t = 1$ and is thus Pareto optimal. It is a matter of routine to check that this allocation cannot be blocked. Any agent heading a blocking coalition needs a strictly positive compensation in the subsequent period. But no matter the type of agent he tries to get into the coalition, the sequence of compensations is strictly increasing and eventually becomes unfeasible.

5. EFFICIENCY, CORE, AND COMPETITIVE EQUILIBRIA

It is quite obvious that any autarkic Pareto optimal competitive equilibrium, i.e., one in which $x_{t,j} = \omega_{t,j}$ for $j = 1, \dots, n$ and $t = 1, 2, \dots$, will belong to the core of the economy. We shall study the relationship between non-autarkic competitive equilibria and the core.

PROPOSITION 2. *Let Assumption 1 be satisfied. Let x be a consumption allocation sequence and p its supporting price sequence. Then if $\text{Lim inf}_{t \rightarrow \infty} \|p^t\| = 0$ the consumption allocation x belongs to the core for every consistent endowments sequence $\omega \in \Omega(x)$.⁷*

⁷ Observe that our result is stronger than Chae's [6] Theorem 4.1, where with a continuum of agents he finds that a sufficient condition for an allocation to belong to the core is that the present value of the total endowment be finite.

Proof. First note that a price sequence with $\text{Lim inf}_{t \rightarrow \infty} \|p^t\| = 0$ cannot correspond to a monetary equilibrium as shown, for instance, in Esteban [9]. Let us now assume that x is blocked by coalition S with consumption allocation \tilde{x} . Without loss of generality we shall assume that $u_{f,j}(\tilde{x}_{f,j}) > u_{f,j}(x_{f,j})$ for some $(f, j) \in S_f$. For any agent $(t, j) \in S_t$, $\tilde{x}_{t,j}$ must not be dispreferred to $x_{t,j}$, so that

$$p^t \cdot [\tilde{x}_{t,j}^t - x_{t,j}^t] + p^{t+1} \cdot [\tilde{x}_{t,j}^{t+1} - x_{t,j}^{t+1}] \geq 0, \quad \text{for all } (t, j) \in S. \quad (1)$$

Since by assumption \tilde{x} is strictly preferred to x by some member of generation f , we have that condition (1) holds as a strict inequality for some agent $(f, j) \in S_f$. From the individual budget constraints we have that

$$p^t \cdot x_{t,j}^t + p^{t+1} \cdot x_{t,j}^{t+1} = p^t \cdot \omega_{t,j}^t + p^{t+1} \cdot \omega_{t,j}^{t+1}, \quad \text{for all } (t, j).$$

Substituting in (1) we obtain

$$p^t \cdot [\tilde{x}_{t,j}^t - \omega_{t,j}^t] + p^{t+1} \cdot [\tilde{x}_{t,j}^{t+1} - \omega_{t,j}^{t+1}] \geq 0 \quad (2)$$

for all $(t, j) \in S$, with strict inequality for some $(f, j) \in S_f$. Adding over all $(t, j) \in S_t$, we have

$$p^t \cdot \sum_{j \in S_t} [\tilde{x}_{t,j}^t - \omega_{t,j}^t] + p^{t+1} \cdot \sum_{j \in S_t} [\tilde{x}_{t,j}^{t+1} - \omega_{t,j}^{t+1}] \geq 0 \quad (3)$$

for $t \geq f$, with strict inequality for $t = f$. Consumption allocation \tilde{x} must be feasible for the coalition members; that is,

$$\sum_{j \in S_{t-1}} [\tilde{x}_{t-1,j}^t - \omega_{t-1,j}^t] + \sum_{j \in S_t} [\tilde{x}_{t,j}^t - \omega_{t,j}^t] = 0 \quad \text{for } t \geq f. \quad (4)$$

Combining (3) and (4) we obtain

$$\begin{aligned} p^{t+1} \cdot \sum_{j \in S_t} [\tilde{x}_{t,j}^{t+1} - \omega_{t,j}^{t+1}] &\geq p^t \cdot \sum_{j \in S_{t-1}} [\tilde{x}_{t-1,j}^t - \omega_{t-1,j}^t] \geq \dots \\ &\geq p^{f+1} \cdot \sum_{j \in S_f} [\tilde{x}_{f,j}^{f+1} - \omega_{f,j}^{f+1}], \quad \text{for } t \geq f+1. \end{aligned} \quad (5)$$

We shall now show that

$$p^{f+1} \cdot \sum_{j \in S_f} [\tilde{x}_{f,j}^{f+1} - \omega_{f,j}^{f+1}] > 0.$$

If $f=0$, this follows from the fact that $\tilde{x}_{0,j}^1$ must be strictly preferred to $x_{0,j}^1$ for some $(0, j) \in S_0$. If $f > 0$ the feasibility condition (4) imposes that

$$\sum_{j \in S_f} [\tilde{x}_{f,j}^f - \omega_{f,j}^f] = 0.$$

Thus, from (3) and bearing in mind that $\tilde{x}_{f,j}$ must be strictly preferred to $x_{f,j}$, the above inequality follows. The terms of the sequence of inequalities (5) can be bounded above by

$$\|p^{t+1}\| \cdot \left\| \sum_{j \in S_t} [\tilde{x}_{t,j}^{t+1} - \omega_{t,j}^{t+1}] \right\| \geq p^{t+1} \cdot \sum_{j \in S_t} [\tilde{x}_{t-1,j}^t - \omega_{t,j}^{t+1}].$$

For economies with $\omega \in \Omega$

$$\left\| \sum_{j \in S_t} [\tilde{x}_{t,j}^{t+1} - \omega_{t,j}^{t+1}] \right\| \text{ is uniformly bounded above.}$$

Hence, if $\text{Lim inf}_{t \rightarrow \infty} \|p^t\| = 0$ it must be that

$$\text{Lim inf}_{t \rightarrow \infty} p^{t+1} \cdot \sum_{j \in S_t} [\tilde{x}_{t,j}^{t+1} - \omega_{t,j}^{t+1}] = 0.$$

Therefore, there does not exist a sequence \tilde{x} satisfying (1) and (4) and x cannot be blocked. Q.E.D.

We shall now study the circumstances under which a competitive equilibrium would not belong to the core of the economy.

PROPOSITION 3. *Let Assumptions 1 and 2 be satisfied. Let x be a consumption allocation sequence with supporting prices p satisfying $\text{Lim inf}_{t \rightarrow \infty} \|p^t\| \geq \varepsilon > 0$. Then there exists $\omega' \in \Omega(x)$ for which x is a competitive equilibrium but does not belong to the core.*

Proof. Let us start by pointing out that for any endowments sequence $\omega \in \Omega$ such that

$$\omega_t^l = x_t^l \quad \text{and} \quad \omega_t^{l+1} = x_t^{l+1}, \quad t = 0, 1, 2, \dots, \tag{6}$$

$$p^t \cdot [x_{t,j}^t - \omega_{t,j}^t] + p^{t+1} \cdot [x_{j,t}^{t+1} - \omega_{t,j}^{t+1}] = 0, \tag{7}$$

$$j = 1, \dots, n \quad \text{and} \quad t = 0, 1, 2, \dots,$$

consumption allocation x will be a competitive equilibrium.

In order to prove the proposition we need to provide an example of a reallocation of initial endowments for which the equilibrium consumption allocation under consideration does not belong to the core. We shall consider in the first place the one-commodity case, i.e., $l = 1$. Moreover, we shall restrict to reallocations of endowments such that $\omega_{t,j} = x_{t,j}$ for $j = 3, 4, \dots, n$ and $t = 1, 2, \dots$, so that the problem is reduced to finding an appropriate distribution of endowments between two agents in each generation, agents 1 and 2.

A clear case of endowment allocation for which x is not in the core is the one satisfying

$$0 < x_{t,1}^{t+1} - \omega_{t,1}^{t+1} \leq \omega_{t+1,1}^{t+1}, \quad t = 1, 2, \dots \tag{8}$$

In that case the coalition formed by the sequence of agents $(1, t), t = 1, 2, \dots$ would block x .

We need now to show that there exists an endowment allocation sequence satisfying (8) as well as (6) and (7). Using (7), (8) can be rewritten as

$$0 < x_{t,1}^{t+1} - \omega_{t,1}^{t+1} \leq (p^{t+2}/p^{t+1})(x_{t+1,1}^{t+2} - \omega_{t+1,1}^{t+2}). \tag{9}$$

Let us construct the endowment sequence ω satisfying (9) such that

$$\omega_{t,j}^{t+1} = x_{j,t}^{t+1} - [\omega_{1,1}^1 - x_{1,1}^1]/p^{t+1}, \quad t = 1, 2, \dots, \quad \text{with } \omega_{1,1}^1 - x_{1,1}^1 > 0. \tag{10}$$

For every value of $\omega_{1,1}^1 > x_{1,1}^1$ and by the use of (10) we can generate a full sequence ω (i.e., $\omega_{t,1}$ and $\omega_{t,2}, t = 1, 2, \dots$) satisfying the equilibrium conditions (6) and (7).

It remains only to verify whether at least one of such sequences of endowments satisfies $\omega \in \Omega$. By assumption 2(c) the sequence x is uniformly bounded from below by some $\lambda > 0$. It is easy to check that for any sequence of endowments obtained from (10), (6), and (7) such that

$$[\omega_{1,1}^1 - x_{1,1}^1]/p^t < \lambda \quad \text{for } t = 1, 2, \dots \tag{11}$$

the consumption allocation x is a Walrasian equilibrium, but does not belong to the core by construction. It is now immediate that whenever $\text{Lim inf}_{t \rightarrow \infty} \|p^t\| \geq \varepsilon > 0$ there exists some $\omega_{1,1}^1$ satisfying (11) such that $\omega_{1,1}^1 > x_{1,1}^1$. This completes the proof for $n = 1$.

The extension to the many commodities case is quite straightforward.

From Assumption 2(b) we have that whenever $\text{Lim inf}_{t \rightarrow \infty} \|p^t\| \geq \varepsilon > 0$, we have that $\text{Lim inf}_{t \rightarrow \infty} p^{t,i} \geq \Pi \cdot \varepsilon > 0, i = 1, \dots, l$. Therefore we can choose $\omega_{t,j} = x_{t,j}$ for $j = 3, 4, \dots, n$ and $t = 1, 2, \dots$ and $\omega_{r,j}^{t+s,i} = x_{r,j}^{t+s,i}$ for $r = 1, 2, s = 1, 2, i = 2, 3, \dots, l$, and $t = 1, 2, \dots$. Then our result for $l = 1$ applies.

Q.E.D.

Proposition 3 can be seen as a source of examples of Pareto optimum competitive equilibria that do not belong to the core of the economy. The example provided in Section 2 of a Pareto optimal competitive equilibrium not in the core is not exceptional.

We have already pointed out in the introduction that for Arrow–Debreu economies the classical set coincides with the Pareto set. Thus the standard

results relating competitive equilibria, efficiency, and core can be restated in terms of the classical set. Specifically, we can say that in Arrow–Debreu economies (i) all the allocations belonging to the classical set belong to the Pareto set (in fact the two sets coincide) and (ii) all the allocations belonging to the classical set belong to the core for all the endowment allocations for which they can be implemented as competitive equilibria. We shall now show that these two propositions hold true for overlapping generations economies once the analysis is restricted to the classical set, i.e., to the set of allocations for which there is no bona fide tax-transfer policy as studied by Balasco and Shell [4, 5].

PROPOSITION 4. *Let Assumptions 1 and 2 be satisfied. Let x be a consumption allocation sequence belonging to the classical set, $x \in \mathcal{C}$. Then x is Pareto optimal.*

Proof. This follows immediately from Proposition 1, taking into account Balasko and Shell's [3] proposition that if the supporting prices of a weakly Pareto optimal consumption allocation satisfy that $\text{Lim inf}_{t \rightarrow \infty} \|p^t\| = 0$ then that consumption allocation is Pareto optimal. Q.E.D.

PROPOSITION 5. *Let Assumptions 1 and 2 be satisfied. Then the consumption allocation sequence x belongs to the core for every $\omega \in \Omega(x)$ if and only if it belongs to the classical set, $x \in \mathcal{C}$.*

Proof. Proposition 1 establishes that the supporting prices of a consumption allocation sequence satisfy $\text{Lim inf}_{t \rightarrow \infty} \|p^t\| = 0$ if and only if x belongs to the classical set. Thus, Propositions 1 and 2 together imply the sufficiency part of this proposition; that is, if a consumption allocation belongs to the classical set it belongs to the core for all endowment sequences for which it is competitive equilibrium. Further, Propositions 1 and 3 together imply the necessity part of Proposition 5. Taken together, they say that if a consumption allocation does not belong to the classical set there exists an endowment allocation for which that consumption allocation does not belong to the core while still being a competitive equilibrium. Q.E.D.

6. MONETARY EQUILIBRIA AND THE CORE

We have already pointed out that overlapping generations models have been considered as the most appropriate framework for the analysis of fiat money. It is thus natural to pay special attention to the relationship between monetary equilibria and core allocations. Besides the obvious relevance of our previous results to monetary equilibria, we shall now

introduce some additional results specifically referring to monetary allocations. We start by demonstrating that monetary equilibria with too much money will not belong to the core. But the main result of this section is that as we enlarge the economy by replication every monetary equilibrium eventually becomes excluded from the core.

With many consumers per generation we may have IOU equilibria, monetary equilibria, and a mixture of the two, i.e., simultaneous use of IOUs and fiat money. The following result refers to economies in which in the long run all intertemporal purchases tend to be made for fiat money. This is an extreme case of a competitive equilibrium in which not only is fiat money the only means of transferring purchasing power from present to the future, but money is present in all transactions. We show that these equilibria do not belong to the core.

PROPOSITION 6. *Let Assumption 1 be satisfied. Let x be a competitive equilibrium consumption allocation sequence and p the sequence of equilibrium prices. Then if*

$$\liminf_{t \rightarrow \infty} p^{t+1} \cdot |x_t^{t+1} - \omega_t^{t+1}| / \mu = 1, \quad \mu > 0, \quad (12)$$

the consumption allocation x does not belong to the core.

Proof. From Definition 5 we know that $p^{t+1} \cdot [x_t^{t+1} - \omega_t^{t+1}] = \mu = \sum_j \mu_j$, $t = 0, 1, 2, \dots$. Therefore, if (12) is satisfied we have that

$$\liminf_{t \rightarrow \infty} p^{t+1} \cdot [x_t^{t+1} - \omega_t^{t+1}] / p^{t+1} \cdot [x_t^{t+1} - \omega_t^{t+1}] = 1.$$

Thus it must be that

$$\begin{aligned} & \liminf_{t \rightarrow \infty} p^{t+1} \cdot [x_{t,j}^{t+1} - \omega_{t,j}^{t+1}] \\ & = \liminf_{t \rightarrow \infty} p^{t+1} \cdot [x_{t,j}^{t+1} - \omega_{t,j}^{t+1}] = z_j \geq z > 0, \quad j = 1, \dots, n, \end{aligned}$$

where $z \in \mathbb{R}_+^l$ and $z(i) = \min\{z_1(i), \dots, z_j(i), \dots, z_n(i)\}$, $i = 1, \dots, l$.

Hence there exist k and λ , $0 < \lambda \leq z$ such that $[x_{t,j}^{t+1} - \omega_{t,j}^{t+1}] \geq \lambda$ for $j = 1, \dots, m$ and $t = k, k + 1, k + 2, \dots$.

Consider now coalition S formed by agents (t, j) , $j = 1, \dots, n$ and $t = k + 1, k + 2, \dots$ with consumption allocation \bar{x} such that

$$\begin{aligned} \bar{x}_{t,j} &= x_{t,j} \quad \text{for all } (t, j) \in S_t, \quad t = k + 2, k + 3, \dots \quad \text{and} \\ \bar{x}_{k+1,j} &= [x_{k+1,j}^{k+1} + \lambda, x_{k+1,j}^{k+2}], \quad j = 1, \dots, n. \end{aligned}$$

Allocation \bar{x} is feasible for coalition S , is as good as x for all members of S , and is strictly preferred by all members of S_{k+1} . Therefore, consumption allocation x does not belong to the core. Q.E.D.

This result seems to substantiate the interpretation given in Esteban [7] to the effect that it is acting as a means of exchange rather than as a store of value that confers social acceptability on fiat money. We have a case in which in the long run all exchanges are intertemporal and made by means of money. Thus, the proposition that no such equilibrium belongs to the core reinforces that view in a many agents economy.

In the following proposition we show that for large economies no monetary equilibrium belongs to the core.

PROPOSITION 7. *Let ξ be an economy with m agents and n goods in which Assumptions 1 and 2(a) are satisfied. Let x be a monetary equilibrium consumption allocation of the economy ξ and p the equilibrium price sequence. Then there exists K such that for the K th replica of the economy $\xi(K)$, the consumption allocation $c(K)$ does not belong to the core.*

Proof. Consider the $(k + 1)$ replica of the economy and let S be a coalition formed by all agents (t, j) , $j = 1, \dots, n$, $t = 1, 2, \dots$, in $\xi(k + 1)$ and all agents $(0, j)$, $j = 1, \dots, n$, in $\xi(k)$. Since x is a monetary equilibrium we have that $p^1 \cdot [\omega_1^1 - x_1^1] = M > 0$. Therefore, there exists at least one vector $\bar{x}_{1,j}^1 \in \mathbb{R}_+^l$ such that $p^1 \cdot [\bar{x}_{1,j}^1 - x_{1,j}^1] > 0$, $j = 1, \dots, n$ and $x_1^1 - \omega_1^1 \leq 0$. Consider now the consumption allocation \bar{x} for the members of coalition S such that $\bar{x}_{t,j} = x_{t,j}$ for all $(t, j) \in S$, $t = 0, 2, 3, \dots$ and $\bar{x}_{1,j} = [(\bar{x}_{1,j}^1 + kx_{1,j}^1)/(k + 1), x_{1,j}^2]$ for all $(1, j) \in S_1$.

Observe that \bar{x} is obtained as a linear convex combination of vectors $x_{1,j}$ and $\bar{x}_{1,j} = [\bar{x}_{1,j}^1, x_{1,j}^2]$ and that as k becomes large $\bar{x}_{1,j} \rightarrow x_{1,j}$. Thus, we have that

$$p^1 \cdot \bar{x}_{1,j}^1 + p^2 \cdot \bar{x}_{1,j}^2 \geq p^1 \cdot x_{1,j}^1 + p^2 \cdot x_{1,j}^2, \quad j = 1, \dots, n.$$

Moreover, it is easy to check that consumption allocation \bar{x} is feasible.

By Assumptions 1 and 2(a) there exists a finite K such that

$$u_{1,j}[(\bar{x}_{1,j}^1 + Kx_{1,j}^1)/(K + 1), x_{1,j}^2] > u_{1,j}[x_{1,j}^1, x_{1,j}^2], \quad j = 1, \dots, n.$$

See Figure 2. Since $\bar{x}_{t,j} = x_{t,j}$ for the rest of the members of S , S will block consumption allocation x . Q.E.D.

This proposition can be interpreted as a formal proof of the observation by Clower that money is held not as the result of individual voluntary choice but by social contrivance, as pointed out in the Introduction.

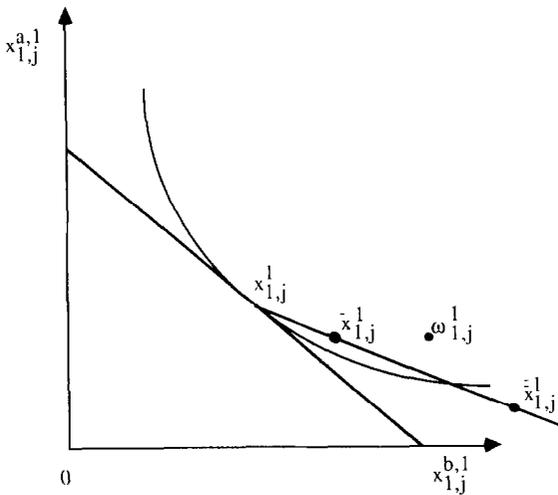


FIGURE 2

7. FINAL REMARKS ON SOME TOPICS IN MONETARY THEORY

We have just demonstrated that for large economies no monetary equilibrium belongs to the core. This result appears to be quite negative with respect to the important literature on fiat money in overlapping generations economies, as developed in Grandmont [14], Kareken and Wallace [16], and Sargent [20], among others. However, it can be interpreted in a more positive spirit as providing a rigorous demonstration of the claim made by Clower [8] that the social acceptance of money is not voluntary and based on its virtue of being a store of value. As a matter of fact, there is nothing terrible or new in this view of money. Douglas Gale [12] and [13], when examining the role of money in the social acceptance of allocations, points out that "it is not the invention of paper money which restores trustworthiness. The Walras allocations are trustworthy in the monetary economy only because there is, in the background, a government which can enforce, evidently at no cost, the payment of [money] taxes. Thus, we have introduced not just a new commodity (money) but a new social institution" (p. 465). From this point of view it is obvious that fiat money has been introduced in overlapping generations models as a commodity and not as a social institution.⁸

Let us be more specific in comparing our results with Douglas Gale's [12] work. As we have already pointed out, he has shown that the intro-

⁸ In this respect, de Vries [21] has recently examined the case in which the acceptance of fiat money is made compulsory, i.e., money is given the status of "legal tender."

duction of money may help in making socially acceptable allocations which would have been blocked without its help. In Gale's model there is a finite number of periods and, if being an Arrow-Debreu economy in every respect, competitive equilibria belong to its core. However, Gale argues that in those equilibria in which there is net borrowing and lending, lenders have good reasons not to trust borrowers. It is in their interest to break futures contracts in later periods of their lives. Gale thus defines the concept of sequential core, i.e., those allocations which belong to the core both in the first and in the subsequent periods, and shows that an allocation is trustworthy if and only if it belongs to the sequential core. Thus, lenders would not trust allocations out of the sequential core even if they belong to the core of the economy. In order to circumvent this problem he proposes that the social institution of money may render otherwise untrustworthy allocations worthy of trust. As in all finite horizon monetary models he needs to introduce a money tax at the end in order to make money valuable. He then finds that in this economy competitive monetary equilibria do belong to the sequential core.

From this point of view, our results can be interpreted in the following way. It seems natural to reconsider his problem in an overlapping generations economy in which one does not need the device of the money tax in order to make money valuable. Then, our findings that Pareto optimal monetary competitive equilibria might not belong to the core and that when the economy becomes large by replication no monetary allocation belongs to the core seem to confirm Gale's assertion that what makes allocations socially stable in his model is not the introduction of fiat money, but the making its use compulsory.

We can go deeper in comparing our model with Douglas Gale's. As we shall now argue, our results can be seen as providing a rationale for Gale's assumption on agents not honoring their contracts. Let us start by noting that an IOU equilibrium in an overlapping generations model can be understood as a sequence of overlapping finite horizon Gale's competitive equilibria. Further, observe that in those equilibria contracts in the futures markets are signed on the two sides by consumers belonging to the same generation. Therefore, as far as futures markets are concerned, IOU equilibria can be seen as a sequence of isolated generations, i.e., as a sequence of overlapping two-period Gale's equilibria. Alternatively, Gale's model can be considered as isolating one single generation of an IOU equilibrium sequence from an overlapping generations model in order to examine its behavior in the futures market.

In spite of their similarity, the two models seem to yield different results. While in Gale's model all competitive equilibria belong to the core, in our model their equivalent, i.e., our IOU equilibria, do not. Thus, the fact of placing a collection of self-contained economies one after the other breaks

the relation between competitive equilibria and core. In other words, from a game-theoretic point of view it makes a substantial difference whether we consider an isolated finite chain of periods or the full infinite sequence. As we have seen in Proposition 5, only those Walrasian equilibria in which there is no borrowing and lending in the long run are always in the core. Hence, the core of an overlapping generations economy formed by chaining a sequence of Gale's two period economies would not contain the allocations that do not belong to the sequential core in Gale's model. Therefore, one need not suppose that agents do not honor their contracts to claim that equilibria which involve borrowing might not be viable. The mere fact that agents live in an endless chain of generations can make IOU equilibria untrustworthy. Our results can thus be considered as a rationale for using the concept of sequential core in finite horizon economies.

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