Social Decision Rules are not Immune to Conflict

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1 Introduction

This short paper asks the following question: why is lobbying so endemic in societies? Put another way, might it not be possible to design a social decision rule that assigns a Pareto-improving outcome to the set of players, by sidestepping the inefficient waste of resources that results from conflict?

We address this question in the particularly simple case of a multi-player *contest*. Each player competes for a prize which is a local public good, worth one util to each winning player. The probability of securing the prize is given by the ratio of player resources devoted to lobbying, to the total resources expended by all of society. There is a disutility cost of supplying effort or conflict resources, which we take to be isoelastic.

Note well that the simplicity of this model is a virtue, not a vice, given that we are interested in the *impossibility* of designing social decision rules.

It is plain that if conflict consumes resources, there are social decision rules that provide Pareto-improvements on the conflictual outcome. For instance, the social decision rule that simply assigns the equilibrium winning probabilities to each parametric configuration of the contest game would surely improve on the equilibrium outcome. However, the computation of these probabilities requires that the social planner know the costs and preferences of the agents involved. We drop this informational stringency in a minimal way, by supposing that the planner has incomplete information *only* regarding the costs of conflict: she knows the winning utilities (which we have normalized to one), she knows the effectiveness of each player, she knows that the cost function is isoelastic, *and* that it is the same across all players. But she does not know the precise value of this elasticity.

We show that this minimal lack of information leads to the impossibility of a

Pareto-improving social decision rule, as long as there are at least four players. For two players it *is* possible to construct such a rule irrespective of the information regarding costs. The case of three players remains open.

The problem addressed has connections with the aggregation of individual preferences over a set of alternatives, as analyzed by social choice theory. We wish to obtain a social decision rule respecting individual preferences. However, we enrich the standard collective choice problem by putting on center stage the description of the non-cooperative outcome that will result in case these individuals fail to agree on a common decision rule. This outcome is the outside option that any player can precipitate. As it turns out, under extremely mild restrictions on the information available, the mere requirement that the outcome of a social decision rule be individually rational with respect to the non-cooperative outcome is sufficient to yield an impossibility result.[1] This is not because the rule fails to satisfy some desirable ethical principle (as in the traditional theory of social choice), but because it cannot guarantee that all players will always prefer the outcome of the rule to the non-cooperative outcome.

The paper is organized as follows. In the next section we develop a simple model of contests with many players. Players are endowed with different capabilities (their abilities to transform effort into success probability). In Section 3 we define *probabilistic social decision rules* (PSDR) and introduce the concept of a PSDR that is *immune to conflict*: a rule with outcomes that Pareto-dominate the equilibrium of the contest game. In Section 4 we prove our result: under mini-

[1] A related approach may be to study the complete information situation and examine the efficiency of particular social decision rules when threat points are subject to strategic manipulation, as in Anbarci, Skaperdas and Syropoulos [1999].

mal restrictions on the information available no PDSR is immune to conflict. We close the paper with some final remarks about the interpretation of our result and provide some numerical examples for the cases of four and five players.

2 A Simple Model of Contests

There are G alternatives and G players. Players have preferences defined over these alternatives. We assume the simplest possible structure on preferences: player i values alternative i by one unit, and places zero valuation on the rest. [For instance, think of the alternatives as potential locations of some public facility with no externalities across locations.]

In the absence of an agreed-upon social decision rule, players see the occurrence of any alternative as probabilistic. Furthermore, they can take (costly) actions that may increase their probability of success. Let us call this activity *lobbying*. We assume that players differ in their lobbying effectiveness. This can be due to differential ability (or means). Denote the relative effectiveness of player i by n_i, with $n(i) > 0$ and $\sum n_i = 1$.[2] Observe that the vector $\mathbf{n} = (n_1, n_2, \ldots, n_G)$ lies in the strictly positive G-dimensional unit simplex.

Let r_i denote the resources contributed by player i. We assume that player i's winning probability $s(i)$ is given by the ratio of player i's effective contribution to

[2]We can also think of players as being groups of different size. In this case, the differential effectiveness of players, n_i, would correspond to the *effective* relative size of a group, with possible rescaling to allow for within-group free-riding. Furthermore, note that in this case we should think of alternatives as local public goods. See Esteban and Ray (1999b) for an analysis of collective action and free-riding in rent-seeking models.

the total (effective) resource contributions in the economy.[3] That is,

$$s_i \equiv \frac{r_i n_i}{\sum_j r_j n_j}.$$

We consider cost functions of the form

$$c(r) = \alpha^{-1} r^{\alpha},$$

where $\alpha > 1$.[4]

Given an anticipated vector of contributions by all other players, player i seeks to maximize (choosing r_i)

$$\frac{r_i n_i}{\sum_j r_j n_j} - \alpha^{-1} r_i^{\alpha}. \tag{1}$$

It follows from (1) and the specification of the cost function that player i's behavior must be described completely by the interior first-order condition

$$\frac{n_i}{R} - \frac{r_i n_i^2}{R^2} = r_i^{\alpha-1},$$

where $R \equiv \sum_i r_i n_i$ denotes the total effective resources expended. With a little bit of manipulation, one obtains a more useful form of the first-order condition, which we record as

$$(\frac{R}{n_i})^{\alpha} = (1 - s_i) s_i^{1-\alpha}. \tag{2}$$

Note that the RHS of (2) is a decreasing function of s_i, so that s_i is defined uniquely for each n_i and is increasing in n_i, for fixed $R > 0$. Think of R as a

[3]See Skaperdas [1996] for an axiomatization of contest success functions. Given the cost functions that we introduce in the text, our success functions fall within this class.

[4]Note that the simplicity of this model is deliberate in that we aim for an impossibility result. For a general model of conflict, see Esteban and Ray [1999a].

scaling factor which then guarantees the equilibrium condition that the shares s_i sum to unity. It is easy to see that there exists a *unique* vector $(s_1, s_2, \ldots, s_G)$ and positive number R that solve (2) for every player $i = 1, ..., G$ and the equilibrium condition that the shares sum to unity.

The equilibrium utility attained by player i is given by,

$$u_i = s_i - \alpha^{-1} r_i^{\alpha} = s_i - \frac{s_i(1 - s_i)}{\alpha}$$

We shall therefore write the vector of equilibrium utilities as $\mathbf{u}(n, \alpha)$.

3 Social Decision Rules

So far we have described how our society might resolve competing interests by taking recourse to conflict. However, conflict consumes resources. Therefore the equilibrium outcome cannot be Pareto-optimal. In what follows we look for a *social decision rule* — a mapping from a domain that represents the planner's information, to allocations — that might create an improvement on the conflictual outcome.

If all relevant information were available to the planner, a social decision rule would map preference profiles, relative power and cost functions into outcomes. In all cases, we allow outcomes to be (possibly degenerate) lotteries over alternatives. To remind the reader of this we will explictly refer to a social decision rule as a PSDR — a "probabilistic" social decision rule.

Given the simplicity of the model, there are only two candidates for possible *exclusion* from the domain — $\mathbf{n}$, the vector of relative powers, and α, the elasticity of the cost function. [Everything else, such as preference structures, are taken to be commonly known.] We impose a minimal amount of ignorance on the planner

by dropping α. That is, we do not allow the planner to condition outcomes on this parameter.

Thus think of a PSDR as a function f that maps every vector of relative powers $\mathbf{n}$ to a lottery over the set of alternatives, which we may identify with the G-dimensional unit simplex with generic element $\mathbf{p}$.

We impose the following minimal condition on any PSDR: it should generate outcomes that (weakly) Pareto dominate the equilibrium utilities in the conflict game. Formally, say that a PSDR f is *immune to conflict* if

$$\mathbf{p} = f(\mathbf{n}) \geq \mathbf{u}(n, \alpha)$$

for all α and all $\mathbf{n}$. [Notice that with our preference normalization, $\mathbf{p}$ is the vector of expected utilities as well as probabilities.] We require, then, that a PSDR be immune to conflict: that no player end up with so much dissatisfaction so as to reject that PSDR in favour of the "anarchic" conflict environment, described in the previous section.

Two remarks are in order. First, observe that even though "immunity to conflict" is fundamentally a behavioral postulate, it has *normative* features as well. We do not mean to suggest that a player, by disagreeing with a rule, can *in fact* precipitate an anarchic society on its own. This would depend on how *other* players might react to this rejection, an issue which is complex and beyond the scope of the current exercise. Rather, we take the anarchic environment as a benchmark and require that organized society should not treat anyone worse than this, as a minimal notion of immunity.

Second, while the current exercise bears a superficial resemblance to the standard implementation problem (see, for instance, Maskin [1985] and Moore [1992]), there are several differences. Perhaps the most obvious of these is our assumption

that the planner makes no attempt (through the design of a suitable mechanism) to elicit the value of α from the players. The question of design is therefore not seriously addressed, and intentionally so.

4 Minimal Lack of Information and the Possibility of Conflict

If there is *complete* information regarding the characteristics of every player, then the domain of any PSDR would include both α and $\mathbf{n}$. Then it is easy to design a PSDR which is both Pareto-optimal and immune to conflict. Simply consider the PSDR that assigns a lottery with probabilities equal to the equilibrium winning probabilities in the conflict game. The objective of this paper is to establish an impossibility result when the assumption of complete information is relaxed in some minimal way. In particular, we show that it is not possible to design a PSDR which is immune to conflict, if the elasticity of the cost function is unknown (though we permit the planner to know that the cost function *is* isoelastic and common to all players).

Proposition **1** *Suppose that there are two players. Then there exists a PSDR which is immune to conflict. Such a PSDR must assign winning probabilities that equal the power share of each player, and it is Pareto-optimal.*

On the other hand, suppose that there are at least four players. Then there exists no PSDR which is immune to conflict.[5]

[5]This possibility result for $G = 2$ also indicates that the negative finding for four or more players is not an intuitive one. Indeed, the case of three players remains open. It is easy to show that, for the larger set of conflict games in which players differ in their valuation of their

Before we present a formal proof, an intuitive discussion may be useful.

We establish the first part of the proposition by observing that regardless of the cost function, a player of power n will have an equilibrium winning probability of precisely n in the conflict game. It is therefore easy enough to design an efficient PSDR which is immune to conflict by simply using relative player power as the winning probability.

That this is the *only* possible rule follows from a general argument for G players (see details in the proof) which establishes that a *necessary condition* for a PSDR be immune to conflict is that the probabilities have to be equal to the relative power. Proving this requires us to vary α over all possible values that exceed unity.[6] As a referee has pointed out, one could alternatively use relative power as a starting point for the analysis. In other words, we could ask whether a PSDR that is proportional to the relative power of players is immune to conflict. This (weaker) question would dispense with the first step of moving α around, and the demonstration that the answer is "no" would coincide with the remainder of our proof.[7]

This remainder involves identifying distributions of power in which the winning probabilities in the non-cooperative game favor one player disproportionately relative to his relative power. When will this happen? Imagine that we start with a

preferred outcome, the impossibility result can be extended to $G = 3$. This being said, we have preferred to present the strongest possible result by obtaining impossiblity even when attention is restricted to a small class of conflict games.

[6]If α is known to have some upper bound, then a PSDR immune to conflict could be drawn from a larger set of PSDRs. We conjecture that if there is such a bound, then — if the number of players is not too high — a possibility result may be recovered. We leave this as an interesting open question.

[7]For axiomatizations of solutions under this interpretation, see Moulin [1999].

distribution of power among two players and substitute the more powerful player by several equal players, while preserving the relative power of the untouched player. In the equilibrium of the conflict game, the winning probability and the expected utility of the untouched player increases as the number of splinter players becomes larger. In fact, as the opponent party is split into more — less powerful — players, the winning probability of the cohesive player approaches arbitrarily close to unity. Then, it will pay the (relatively) most powerful player to reject the application of the proportional rule.

We now turn to a formal account.

Proof of Proposition 1. Take any power distribution $\mathbf{n} \gg 0$ over G. For a given cost elasticity α, denote by $\{s_i(\alpha), r_i(\alpha)\}_{i \in G}$ the (unique) equilibrium shares and individual costs under this situation. Slightly rewriting the first-order conditions (2) characterizing equilibrium conflict, we see that for every player i,

$$s_i(\alpha)[1 - s_i(\alpha)] = r_i(\alpha)^\alpha. \tag{3}$$

Now consider some sequence $\alpha \to \infty$. Because the left-hand side of (3) is bounded in α, it follows that $\frac{1}{\alpha} r_i(\alpha)^\alpha \to 0$ as $\alpha \to \infty$.

Denote by $\{s_i^*, r_i^*\}_{i \in G}$ any limit point of $\{s_i(\alpha), r_i(\alpha)\}_{i \in G}$ as $\alpha \to \infty$. To save on notation, let $\{\alpha\}$ itself denote the subsequence along which the equilibria converge to this limit point. By the observation in the previous paragraph, we see that the limit utilities u_i^* are given by

$$u_i^* \equiv \lim_{\alpha \to \infty} [s_i(\alpha) - \frac{1}{\alpha} r_i(\alpha)^\alpha] = s_i^* \tag{4}$$

It follows from (4) that for a rule f to be immune to conflict under the population distribution $\mathbf{n}$ on G, it must be that $f_i(\mathbf{n}) \geq s_i^*$ for every $i \in G$. But, because

$\sum_{i \in G} s_i^* = 1$, this implies that

$$f_i(\mathbf{n}) = s_i^* \tag{5}$$

for each $i \in G$. Now consider two cases.

Case 1. *For some i, $s_i^* = 0$.* In this case, it follows from (5) that the rule cannot be immune to conflict, because we know that for this player i and any $\alpha > 1$, equilibrium utility is strictly positive.

Case 2. *For every i, $s_i^* > 0$.* In this case, we claim first that $s_i^* = n_i$ for all $i \in G$. To establish this, notice that if $1 > s_i^* > 0$ for any i, $r_i^* = 1$. This is because in such a case, the left hand side of (3) is positive and bounded away from zero in α. This can only happen if the limit value r_i^* equals unity.

To complete the proof of the claim, we observe that because $r_i^* = 1$ for all i, the limiting value of R — call it $R^* \equiv \sum_{i \in G} n_i r_i^*$ — equals unity as well. Consequently,

$$s_i^* = \frac{n_i r_i^*}{R^*} = n_i \tag{6}$$

for all $i \in G$.

Combining (5) and (6), it follows that

Any rule f which is immune to conflict must satisfy $f_i(\mathbf{n}) = n_i$ for all $i \in G$ and $\mathbf{n}$.

When $G = 2$, one can easily use (2) to conclude that in equilibrium, $s_i = n_i$ for all $i \in G$. Moreover, because $r_i > 0$,

$$u_i = s_i - \frac{1}{\alpha}(r_i) < s_i = n_i,$$

so that the only possible candidate rule f — this is the one assigning probabilities according to $f_i(\mathbf{n}) = n_i$ for all i — is immune to conflict. This establishes the first part of the proposition.

To continue in the case of more players, rewrite (2) after some manipulation as

$$n_i = R\left(\frac{s_i^{\alpha-1}}{1-s_i}\right)^{1/\alpha}, \tag{7}$$

so that adding over all n_i we obtain

$$\frac{1}{R} = \sum_{i=1}^{G}\left(\frac{s_i^{\alpha-1}}{1-s_i}\right)^{1/\alpha}.$$

Substituting this in (7), we see that

$$n_i = \frac{\left(\frac{s_i^{\alpha-1}}{1-s_i}\right)^{1/\alpha}}{\sum_{i=1}^{G}\left(\frac{s_j^{\alpha-1}}{1-s_j}\right)^{1/\alpha}} \tag{8}$$

Equation (8) tells us the population vector that generates some given vector $\mathbf{s}$ as the equilibrium share vector.

Consider the special share vector given by $s_1 = s$ and $s_i = \frac{1-s}{G-1}$ for $i \neq 1$, defined for any $s \in (0,1)$. Substituting this expression in (8) and simplifying, we see that

$$n_1 = \frac{1}{1+(G-1)^{2/\alpha}s^{(1-\alpha)/\alpha}(1-s)(G-2+s)^{-(1/\alpha)}}. \tag{9}$$

Now observe that the equilibrium utility for player 1 is given by

$$u_1 \equiv s - \frac{r^\alpha}{\alpha} = s - \frac{s(1-s)}{\alpha}$$

which, used in (9) tells us that

$$u_1 - n_1 = \frac{(1-s)\{(G-1)^{2/\alpha}s^{1/\alpha}(\alpha-1+s)(G-2+s)^{-(1/\alpha)} - \alpha - s\}}{1+\alpha\{(G-1)^{2/\alpha}s^{(1-\alpha)/\alpha}(1-s)(G-2+s)^{-(1/\alpha)}\}}. \tag{10}$$

The sign of (10) is determined simply by the sign of the numerator. Evaluating the limit of this numerator as $s \uparrow 1$, we see that it is

$$\alpha(G-1)^{1/\alpha} - (1+\alpha)$$

which is certainly strictly positive for some $\alpha \in (1, \infty)$, provided that $G \geq 4$. [For instance, take α close to 1 or close to 2.]

It follows that whenever $G \geq 4$, then the corresponding equilibrium utility for player 1 under the associated population vector *strictly exceeds* the population share of that player for some values of $\alpha \in (1, \infty)$.

On the other hand, we have already seen that in the present case, $f_i(\mathbf{n}) = n_i$ for all population vectors $\mathbf{n}$, if it is to be immune to conflict. What we have just established is that such a rule isn't, in fact, immune, so the proof of the proposition is complete.

5 Concluding Remarks

In this paper we address the problem of aggregation of individual preferences into a PSDR in the context of a conflict game between players with opposing interests and varying power. We have shown that, for $G \geq 4$ and under a minimal restriction on the information available, there is no PSDR that weakly Pareto dominates the equilibrium of the conflict game and is thus immune to conflict.

The proof of the result consists of two steps. First we show that all PSDR *must* assign probabilities equal to the relative power of the parties. The second step consists of showing that for $G \geq 4$ the returns to power are sufficiently strong to more than compensate for the saving in the resources expended in conflict. Rather than increasing the power of one group at the expense of the others, we keep the relative power of one player constant and divide up the remaining power over an increasing number of players. As it turns out, when the opposition is sufficiently divided up the winning probability of the untouched player in the conflict game

can be made arbitrarily close to unity.[8]

Does this require that the powerful player face an extremely large number of powerless players? Not quite. Consider the following numerical illustrations for $\alpha = 2$. With $G = 5$, the player with $1/3$ of the power will strictly prefer the conflict outcome if it faces four players of power $1/6$. The same is true for $G = 4$ when a player with $1/2$ faces three players of equal power, $1/6$. These are not large numbers by any stretch.

It is worth noticing that for these skewed distributions the corresponding equilibrium level of conflict is low. Indeed, in the situations in which refusing a PSDR carries high costs in terms of conflict, players are prepared to accept larger deviations between the probabilities assigned by the PSDR and the conflict outcome. However, in the situations in which the conflictual resolution of opposing interests is not very costly, players will switch more easily to the conflict mode and PSDR rules will be more easily rejected by some party.

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[8]Notice that this result seems very much in line with the classical motto "divide and rule".

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