J Econ Inequal
DOI 10.1007/s10888-006-9032-x

An Extension of a Measure of Polarization, with an application to the income distribution of five OECD countries

Joan Esteban · Carlos Gradín · Debraj Ray

Received: 9 July 2005 / Accepted: 9 May 2006

Abstract We introduce an extension of the Esteban and Ray [Econometrica, 62:819–851 1994] measure of polarization that can be applied to density functions. As a by-product we also derive the Wolfson [Am. Econ. Rev., 84:353–358 1994] measure as a special case. This derivation has the virtue of casting both measures in the context of a (statistically) unified framework. We study the polarization of the distribution of household income for five OECD countries (LIS database): US, UK, Canada, Germany and Sweden.

Key words income distribution · inequality · polarization.

JEL classification D31 · D63 · I32.

1. Introduction

The concept and measurement of the polarization of a distribution has recently attracted some attention from economists. In independent work, Esteban and Ray [8] (henceforth, ER) and Wolfson [16] (henceforth, W) have developed measures

The present paper is essentially based on Esteban, Gradín and Ray [9], published as #218 in the working papers series of the Luxembourg Income Study. Changes have been kept to a minimum. We have provided a brief reference to the literature on polarization measurement that has appeared since then and have updated the data of our application.

J. Esteban (✉) · D. Ray
Institut d'Analisi Económica, CSIC, Barcelona, Spain
e-mail: joan.esteban@uab.es

C. Gradín
Universidade de Vigo, Vigo, Spain

D. Ray
New York University, New York, NY, USA

of polarization that, while sharing similar motivations, turn out to have significant differences.[1] Both the ER and W measures have been extended and/or applied in different contexts; see, for instance, Wang and Tsui [15], Zhang and Kanbur [17], Chakravarty and Majumder [4, 5], Salas and Rodríguez [14], Anderson [1, 2] and Montalvo and Reynal-Querol [12].

The ER formulation relies fundamentally on what we have called elsewhere the *identity-alienation* framework (see Duclos et al. [7]). The framework, which we discuss briefly in the next section, relies on the presumption that individuals are 'identified' with others who are 'close' to them, while they are 'alienated' from others who are 'far away'. The ER measure *assumes* that the data arrives pre-grouped into the appropriate population clusters, within which there are bonds of identification and across which are the tensions of alienation. But of course, the *statistical* classes into which the distributional data may be grouped (e.g., deciles) may have nothing to do with the former conceptual grouping. More pertinently (and increasingly more often), income distributions may be presented in the form of some density function, which has been estimated from sample data using parametric or nonparametric methods.

To be sure, ER discuss these issues and suggest an extension of the measure to deal with them. The purpose of this paper is to examine a particular extension in detail.

One possible approach would be to approximate the strength of group identification for a person by using the value of the density function evaluated at that person's income. This is the method followed (and discussed) in Duclos, Esteban and Ray [7] in their exploration of ER for the case of continuous distributions. The purely statistical approach we take here, while complementary, is somewhat different. Suppose that a fixed number of income cutoffs are given to the researcher (e.g., cutoffs to determine low, middle-class and high income groups). These pin down the number of groupings but not their locations. We propose to pin down group locations (or equivalently, the positioning of the cutoffs in income space) by minimizing the dispersions within the clusters created by any given number of income cutoffs. We then apply the ER measure to the discrete groupings thus created, with a correction for intra-group dispersions. This yields an extended measure of polarization which can be applied to all sorts of income distributions, especially when they are in the form of densities.

A serendipitous byproduct of our approach is that we derive the W measure as a particular special case of this formulation. Thus our derivation has the virtue of casting both the ER and W measures in the context of a (statistically) unified framework.

In Section 2 we set the stage by describing briefly the identity-alienation framework. In Section 3, we describe our extended measure, drawing on a 'statistical' approach. We also examine the properties of this second type of extension for the special case of bi-polarization and derive Wolfson's measure as a particular case. In Section 4, we study the polarization of the distribution of household income for five OECD countries (LIS database).

[1] We are grateful to James Foster for bringing to our attention, after ER was published, the existence of Love and Wolfson [11], in which similar concerns with the Pigou–Dalton principle of transfers were first raised.

2. Conceptual issues

Suppose that F is some (estimated or true) income distribution. As in ER, we will suppose that each individual is subject to two forces: She feels *identification* with those she considers to be members of her 'own group', and alienation from those she considers to be members of 'other groups'. Thus, keeping matters deliberately abstract for the moment, suppose that an individual with income x feels group identification $I(x, F)$ under the distribution F, and alienation $r(x, y)$ with respect to some individual with income y. As in ER, we take the *effective antagonism* that individual x feels towards y as some function $T(I, r)$ strictly increasing in r. Effective antagonism increases with alienation, but this alienation is taken to be fueled by some sense of identification as well.

Polarization is the 'sum' of all effective antagonisms:

$$P(F) = \iint T(I(x,F), r(x,y))\mathrm{d}F(x)\mathrm{d}F(y) \quad (1)$$

Described in this way, the measure is not very operational. Much is obviously left to the choice of the identification and alienation functions, as well as the function T that aggregates them to precipitate a measure of polarization. The approach taken in ER is to combine this relatively broad starting point with a set of intuitive axioms that might compare polarization across distributions. These axioms yield restrictions on the functional forms that can be admitted into the general framework sketched in Esteban and Ray [8]. The ER characterization is restricted, however, to distributions that are pre-arranged in groups so that for an individual with income x belonging to some group i, $I(x, F)$ simply equals p_i, the proportion of individuals in that group (under the distribution F).

But there is, of course, no reason to believe that the grouping of income distribution data will conveniently conform to the psychological demands of group identification. Consider, therefore, the following extension along the lines suggested by ER: Let $D > 0$ be such that if an income y' is within D of an income y, then there is some identification between the two incomes. More formally, let $w(d)$ be a positive weighting scheme on $[0, D]$ such that $w(.)$ is a decreasing function with $w(D) = 0$. For a given distribution F, define the identification felt by any individual with income x as

$$I(x,F) = \int_{\{y:|y-x|\le D\}} w(|y-x|)\mathrm{d}F(y) \quad (2)$$

Take alienation to be simply the linear distance between x and y, with the identification zone netted out:

$$r(x,y) = \max\{|x-y| - D, 0\}. \quad (3)$$

Then, a natural generalization of the ER measure is

$$P(\alpha,F) = \iint \left[\int_{y:|y-x|\le D} w(|y-x|)\mathrm{d}F(y) \right]^{\alpha} \max\{|x-z| - D, 0\}\mathrm{d}F(x)\mathrm{d}F(z), \quad (4)$$

where α is some positive constant capturing the importance of group identification in the determination of interpersonal effective antagonism.

Observe that if group identification is unimportant, then we can take $D = 0$ and $\alpha = 0$ as well, in which case the measure in (3) reduces to a measure of *inequality*, the Gini coefficient. Thus it is the presence of identification that makes a measure of polarization fundamentally different from one of inequality (for more on this, see ER).

We need to be aware of some features of (4). First, if the distribution is clustered entirely within a support of D, then polarization is zero. This is not a problem at all provided we do not insist that any difference in incomes, however slight, should result in some polarization. Second, and more problematic, is the fact that the measure is still not operational: How does one choose the weighting function, or indeed the domain of identification D? It is clear that, beyond a point, it is difficult to nail these objects down: One can only hope for robustness of the implied polarization ordering under various choices of the functional forms.

In the next section, we take a particular approach to this question. To be sure, no one method can get around the problems raised here, but the measure that we obtain has the virtues that (a) it does not disagree with the overall conceptual scheme presented in this section, and (b) it is tractable and easy to implement.

Before we proceed, it should be noted that we presume throughout that income similarities or differences form the basis for identification or alienation. This assumption may well be wrong: Religion, kin, language or occupation may be the more salient characteristic. In such cases, one may wish to develop a parallel measure of polarization for each of these characteristics, as in Gradín [10] or Reynal-Querol [13]. Alternatively, it may be possible to work towards a multidimensional, hybrid notion of polarization, as in Duclos et al. [7]. In any case, we do not go into such issues here.

3. A 'statistical' approach

The view we develop in this extension of the ER measure of polarization can be summarized as follows. The ER polarization measure for discrete groups should be used only *after* the population has been regrouped in a way that captures the group identification structure of society. This regrouping or clustering will lose some of the initial information that concerns the dispersion of the population around the clusters that we are treating as single groups. Put another way, the artificial sharpness of identification induced by the ER measure needs to be corrected for. From a 'statistical' perspective, the clustered data will contain some degree of error relative to the original information. The extended measure we propose is the polarization measure (as in Esteban and Ray [8, equation 3]) on the clustered distribution corrected by a measure of the error made by clustering.

3.1. The general case

Suppose that income distribution data are provided by means of a density f. Let the support of the distribution be contained in some bounded interval $[a, b]$. Incomes are normalized to the expected income, $\mu = 1$. An *n-spike representation* of f is a collection ρ of numbers (y_0, y_1,..., y_n; π_1,..., π_n; μ_1,..., μ_n) such that $a = y_0 <...< y_n = b$, and

$$\left.\begin{aligned} \pi_i &= \int_{y_{i-1}}^{y_i} f(y)\mathrm{d}y, \\ \mu_i &= \frac{1}{\pi_i}\int_{y_{i-1}}^{y_i} yf(y)\mathrm{d}y \end{aligned}\right\} \text{, for all } i = 1, ..., n. \tag{5}$$

Each n-spike representation ρ of f induces an approximation error, which we denote by $\varepsilon(f, \rho)$. The error corresponds, as we have already discussed, to the implicit fuzziness of group identification: After all, the spikes are only a representation.

Looking ahead, we are going to define our measure of extended polarization in the following way:

$$P(f; \alpha, \beta) = \mathrm{ER}(\alpha, \rho) - \beta\varepsilon(f, \rho), \tag{6}$$

where $\mathrm{ER}(\alpha, \rho)$ is the ER measure of polarization with parameter α applied to the n-spike representation ρ, given by

$$\mathrm{ER}(\alpha, \rho) = \sum_i \sum_j \pi_i^{1+\alpha}\pi_j \left|\mu_i - \mu_j\right| \tag{7}$$

and where β is a free parameter which measures the weight we attach to the 'measurement error' (or lack of identification) in downscaling the ER polarization computed from the representation. Since the ER measure is defined on the simplified representation of the distribution, we shall refer to it as 'simple' polarization and use the term 'extended' polarization for the measure on the complete distribution.

Note that the measure described in (6) is not a special case of the class of measures defined by (4), simply because we subtract any fuzziness in identification linearly and do not interact it with the alienation term. It may well be that the correction applied should depend on the amount of alienation felt by individuals in that group: These would give rise to interesting alternative measures after the 'correction' is applied. We consider here the simplest possible case which respects the broad conceptual issues raised in Section 2.

Now we turn to the question of an appropriate n-spike representation for the income distribution at hand. There are really two questions here: One has to do with the *number* of spikes involved in the representation, and the other has to do with their *locations*. We view the number as exogenous (for instance, standard economic categories may use the 'poor', the 'middle class', and the 'rich'), but concentrate on the endogenous determination of the locations. To be sure, there is no single answer to this question, but it is clear that the locations should respect some notion of group identification: A *group*, represented by a typical interval of the form $[y_{i-1}, y_i]$, should not have a large dispersion in the characteristics of its members (relative to the dispersion in the overall distribution).

One way, then, to locate the spikes is to define the approximation error $\varepsilon(f, \rho)$ as

$$\varepsilon(f, \rho) = \frac{1}{2}\sum_i \int_{y_{i-1}}^{y_i}\int_{y_{i-1}}^{y_i} |x - z| f(x) f(z)\mathrm{d}x\mathrm{d}z \tag{8}$$

and choose the approximation ρ (for given n) that minimizes this error. In this way one minimizes the average difference of income pairs within the groups, that is the average within group alienation. Thus, implicitly, the dispersion within each group is being measured by the Gini coefficient. This is precisely the approach taken by Aghevli and Mehran [3] and Davies and Shorrocks [6].

Let ρ^* be the n-spike representation that solves this problem. This solution is characterized by the condition that

$$y_i^* \int_{y_{i-1}^*}^{y_{i+1}^*} f(x)\mathrm{d}x = \int_{y_{i-1}^*}^{y_{i+1}^*} xf(x)\mathrm{d}x, \quad \text{for} \quad i = 1, ..., n-1. \tag{9}$$

Expression (9) has quite a simple interpretation. Using our notation it may be rewritten as

$$y_i^* = \frac{\pi_i^* \mu_i^* + \pi_{i+1}^* \mu_{i+1}^*}{\pi_i^* + \pi_{i+1}^*}. \tag{10}$$

That is, the dividing income between any two adjacent intervals has to be equal to the average income of these two intervals taken together.

Diagrammatically, an n-spike representation of F is equivalent to transforming the original Lorenz curve into a piecewise linear Lorenz curve (with n pieces). Hence, the minimization of (8) is equivalent to minimizing the area between the original Lorenz curve and the piecewise linear representation, as Figure 1 shows. It is therefore immediate that

$$\varepsilon\left(f, \rho^*\right) = G(f) - G\left(\rho^*\right), \tag{11}$$

where $G(.)$ assigns the Gini coefficient to the distribution variable in its argument.[2] Combining Equations (6) and (11), we see that

$$P(f, \alpha, \beta) = \mathrm{ER}\left(\alpha, \rho^*\right) - \beta\left[G(f) - G\left(\rho^*\right)\right] \tag{12}$$

This is the proposed extended polarization measure.

3.2. The case of bi-polarization

Of special interest is the question: How *bipolar* is society? How close is the distribution to the formation of two large groups, presumably identified within each and

[2] Note now that any ρ induces a partition of the support of f into non-overlapping intervals. For mean income normalized to unity, the Gini coefficient of f, $G(f)$, can be written as

$$G(f) = \iint |x - z| f(x) f(z)\mathrm{d}x\mathrm{d}z = \frac{1}{2}\sum_i \int_{y_{i-1}}^{y_i} \int_a^b |x - z| f(x) f(z)\mathrm{d}x\mathrm{d}z$$

$$= \frac{1}{2}\sum_i \int_{y_{i-1}}^{y_i} \int_{y_{i-1}}^{y_i} |x - z| f(x) f(z)\mathrm{d}x\mathrm{d}z + \frac{1}{2}\sum_i \sum_j |\mu_i - \mu_j| \pi_i \pi_j$$

$$= \varepsilon(f, \rho) + G(\rho).$$

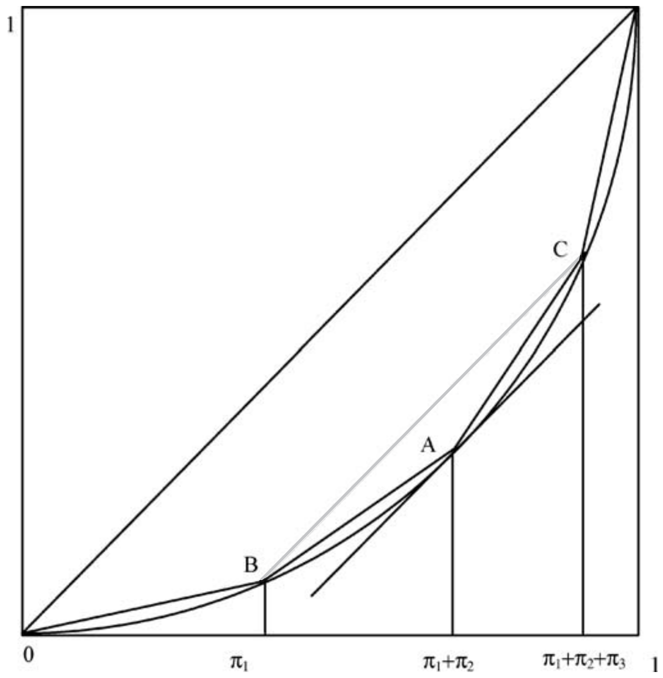

Figure 1 Error-minimizing *n*-spike representation ($n = 4$).

standing in antagonism to each other? This kind of question can be addressed by taking $n = 2$ in the exercise above. As a matter of fact, this might not be an accurate way of capturing the social groupings that actually do exist. Thus this question is different from the issue of how *polarized* a distribution actually is, but it is a perfectly legitimate handle on the related concept that we shall refer to as *bipolarization*.

It will be enough in this case to focus on y_1, the cut-off that divides the two presumed groups. We shall drop subscripts here and refer to this cut-off as y, while π will denote the value of the cumulative distribution up to y: That is,

$$\pi = \int_a^y f(x)\mathrm{d}x. \tag{13}$$

Normalize so that mean income for the entire distribution equals unity. Let $L(\pi)$ denote the ordinate of the Lorenz curve of f at the point π. Then it is easy to check that $\mu_1 = L(\pi)/\pi$ and $\mu_2 = [1 - L(\pi)]/[1 - \pi]$. Consequently,

$$\begin{aligned} \mathrm{ER}(\alpha, \rho) &= \left\lfloor \pi^{1+\alpha}(1-\pi) + (1-\pi)^{1+\alpha}\pi \right\rfloor [\mu_2 - \mu_1] \\ &= \left[\pi^{\alpha} + (1-\pi)^{\alpha}\right][\pi - L(\pi)] \end{aligned} \tag{14}$$

while

$$\varepsilon(f, \rho) = G - [\pi - L(\pi)]. \tag{15}$$

Combining (14) and (15) in the way indicated by (6), but not optimizing with respect to the error just yet, we see that

$$P(f;\alpha,\beta,y) = \left[\pi^{\alpha} + (1-\pi)^{\alpha}\right][\pi - L(\pi)] - \beta\{G - [\pi - L(\pi)]\}, \tag{16}$$

where the inclusion of y inside the P-function reminds us that we have not yet optimized with respect to the cut-off income level.

Such optimization clearly means that we choose y, or equivalently π, to maximize the vertical difference between the Lorenz curve and the 45° line:

$$\max_{\pi}[\pi - L(\pi)].$$

If the Lorenz curve is strictly convex (as it typically will be if the data is presented in the form of a density), there is a unique solution to this problem, $y = \mu$. Taking into account that the mean deviation D is

$$D = \frac{1}{2\mu}\int |\mu - y| f(y)\mathrm{d}y = \pi_{\mu} - L\big(\pi_{\mu}\big), \tag{17}$$

where $\pi_{\mu} = F(\mu)$. Bearing in mind that $\mu = 1$, we may rewrite (16) as

$$P(f;\alpha,\beta) = \left[\pi_{\mu}^{\alpha} + \big(1-\pi_{\mu}\big)^{\alpha}\right]D - \beta(G - D) \tag{18}$$

Note finally that for α =1 we have the following simple expression

$$P(f;1,\beta) = (1+\beta)D - \beta G \tag{19}$$

But (16) also holds a route to the measure of polarization proposed by Wolfson [16]. To see this, suppose that we choose y to equal the median income, call it m. Then $\pi = 1/2$, so that (16) becomes

$$P(f;\alpha,\beta,m) = \big(2^{1-\alpha} + \beta\big)\left[\frac{1}{2} - L\left(\frac{1}{2}\right)\right] - \beta G. \tag{20}$$

If we specialize further to the case in which $\alpha = \beta = 1$, then we have

$$P(f;1,1,m) = 2\left[\frac{1}{2} - L\left(\frac{1}{2}\right)\right] - G = \frac{m}{2}P^{\mathrm{W}}(f), \tag{21}$$

where $P^{\mathrm{W}}(f)$ stands for the W measure of polarization.

We conclude this section with a word of caution on the use of measures of bipolarization such as (21). As we have seen, the measure not only presumes that there are two groups, it uses the median as the demarcating line between the two groups, instead of going through the error-minimizing process described earlier in (8)–(10). The exogenous use of the median as a cutoff may lead to some counterintuitive observations. As an example, start with a distribution on nk equally sized spikes. Suppose that we concentrate the population of each k adjacent spikes and generate a new distribution into n equidistant equally sized spikes. We now wish to compare the degree of bipolarization before and after the concentration has taken place. Then, this concentration of probability will imply an increase or decrease in bipolarization, as measured by (21), according to whether n is even or odd.[3] The reason is simply that, departing from the uniform distribution, an even number of spikes takes us closer to the symmetric bipolar distribution, while an odd number takes us farther away. In contrast, the ER measure of polarization will always increase, as shown in Esteban and Ray [8, Section 3.4.3].

3.3. Concluding remarks on the extended measure

In this section, we show how the ER polarization measure can be extended to cover situations in which the primitive distributional data are captured by a density, or by some grouping that does not correspond naturally to the notion of group identification. A by-product of this extension is that it provides an independent route, via the ER measure, to the W measure of polarization. In this way, we attempt to provide both an extension of existing literature on polarization measurement and a synthesis of it.

4. Income polarization in five OECD countries

Using the approach developed above we shall now analyze the level of polarization in the size distribution of household incomes in five selected OECD countries: Canada, Germany, Sweden, the UK and the US, for the period 1974–2000. We shall use the Luxembourg Income Study database (LIS), which furnishes standardized household income distributions for a number of countries, permitting meaningful international comparisons. We shall thus work with household disposable income as defined in the LIS (household yearly earnings + cash property income + social insurance + social transfers − tax − mandatory contributions) adjusted according to the OECD equivalence scales. The cost of using this data set is that we are faced with severe limitations on the countries and years covered. This is the reason why we use such a limited number of countries. But, even then, there is not a single year for which we have information for the five countries at a time.

We compute the extended polarization measure for two, three and four groups and for different values of the sensitivity parameter α (1, 1.3, and 1.6). Furthermore, we take $\beta = 1$ throughout. Extended polarization is reported in Table I. Table II presents the data for simple polarization and Table III the intra-group dispersion. Table IV provides the decomposition of the within-group dispersion by income classes (two, three and four). Tables V, VI and VII furnish supplementary information on the inter-group cut-off incomes, the group population shares, and the within-group mean incomes.

Let us start by focusing on the case of three groups and α=1. By 2000, at the end of the period considered, we find that the US and the UK are the most polarized countries. They are more polarized than Sweden, the least polarized of the

[3] We make these computations by leaving a mass ε at the median income and letting ε go to zero.

Table I Extended polarization $P(f, \alpha, \beta = 1)$

N^{er} of groups	$P(f, \alpha, =1, \beta = 1)$			$P(f, \alpha, =1.3, \beta = 1)$			$P(f, \alpha, =1.6, \beta = 1)$			Gini
	2	3	4	2	3	4	2	3	4	
US										
1974	0.1410	0.1485	0.1289	0.0965	0.0945	0.0767	0.0605	0.0558	0.0422	0.3478
1979	0.1381	0.1439	0.1242	0.0949	0.0917	0.0736	0.0599	0.0544	0.0404	0.3311
1986	0.1507	0.1553	0.1342	0.1044	0.0994	0.0801	0.0670	0.0593	0.0444	0.3563
1991	0.1524	0.1570	0.1355	0.1059	0.1005	0.0811	0.0684	0.0601	0.0452	0.3590
1994	0.1619	0.1670	0.1440	0.1126	0.1074	0.0863	0.0730	0.0646	0.0482	0.3841
1997	0.1605	0.1683	0.1468	0.1119	0.1092	0.0893	0.0729	0.0668	0.0512	0.3901
2000	0.1573	0.1659	0.1441	0.1094	0.1075	0.0873	0.0709	0.0656	0.0496	0.3841
UK										
1974	0.1226	0.1252	0.1090	0.0857	0.0806	0.0659	0.0560	0.0487	0.0374	0.2871
1979	0.1227	0.1231	0.1053	0.0863	0.0794	0.0633	0.0570	0.0481	0.0355	0.2769
1986	0.1276	0.1296	0.1118	0.0885	0.0823	0.0660	0.0571	0.0485	0.0358	0.3078
1991	0.1526	0.1544	0.1335	0.1079	0.1007	0.0816	0.0720	0.0622	0.0471	0.3495
1995	0.1471	0.1503	0.1282	0.1033	0.0976	0.0767	0.0682	0.0597	0.0427	0.3462
1994	0.1483	0.1525	0.1301	0.1045	0.0997	0.0785	0.0696	0.0617	0.0444	0.3519
1999	0.1516	0.1562	0.1352	0.1062	0.1012	0.0816	0.0700	0.0617	0.0460	0.3660
Sweden										
1975	0.1000	0.0996	0.0862	0.0689	0.0627	0.0505	0.0437	0.0361	0.0270	0.2331
1981	0.0803	0.0835	0.0733	0.0540	0.0517	0.0427	0.0327	0.0290	0.0225	0.2011
1987	0.0903	0.0955	0.0849	0.0597	0.0586	0.0493	0.0350	0.0322	0.0260	0.2359
1992	0.0925	0.0987	0.0868	0.0616	0.0611	0.0505	0.0364	0.0343	0.0267	0.2413
1995	0.0835	0.0945	0.0845	0.0539	0.0582	0.0494	0.0298	0.0322	0.0263	0.2353
2000	0.0993	0.1054	0.0927	0.0668	0.0660	0.0545	0.0406	0.0377	0.0293	0.2571
Germany										
1973	0.1170	0.1220	0.1063	0.0805	0.0776	0.0633	0.0512	0.0457	0.0349	0.2877
1978	0.1134	0.1184	0.1030	0.0781	0.0756	0.0615	0.0498	0.0450	0.0341	0.2780
1981	0.1097	0.1127	0.0978	0.0755	0.0717	0.0582	0.0479	0.0423	0.0321	0.2629
1983	0.1099	0.1157	0.1000	0.0759	0.0742	0.0598	0.0486	0.0446	0.0333	0.2690
1984	0.1119	0.1204	0.1063	0.0746	0.0752	0.0623	0.0445	0.0429	0.0333	0.2983
1989	0.1061	0.1143	0.0995	0.0715	0.0723	0.0588	0.0437	0.0422	0.0319	0.2739
1994	0.1134	0.1216	0.1048	0.0772	0.0774	0.0620	0.0480	0.0457	0.0338	0.2869
2000	0.1106	0.1156	0.1015	0.0756	0.0729	0.0602	0.0474	0.0423	0.0330	0.2754
Canada										
1975	0.1348	0.1404	0.1213	0.0924	0.0893	0.0719	0.0581	0.0526	0.0393	0.3250
1981	0.1324	0.1350	0.1168	0.0917	0.0859	0.0694	0.0588	0.0508	0.0382	0.3122
1987	0.1299	0.1323	0.1141	0.0901	0.0843	0.0678	0.0580	0.0499	0.0372	0.3066
1991	0.1253	0.1278	0.1105	0.0865	0.0810	0.0653	0.0552	0.0475	0.0355	0.2992
1994	0.1294	0.1302	0.1128	0.0900	0.0825	0.0668	0.0582	0.0485	0.0366	0.3036
1997	0.1260	0.1296	0.1120	0.0870	0.0824	0.0663	0.0555	0.0486	0.0363	0.3026
1998	0.1312	0.1364	0.1179	0.0901	0.0869	0.0699	0.0570	0.0514	0.0382	0.3210
2000	0.1252	0.1308	0.1145	0.0858	0.0828	0.0681	0.0541	0.0484	0.0375	0.3107

Source for all tables and figures: Own construction based on LIS database. Original data for US (CPS), UK (1974–1991 and 1995, FES; 1994 and 1999, FRS); Sweden (IDS); Germany (1973–1983, EVS; 1984–2000, GSOEP); Canada (1971–1997, SCF; 1998–2000, SLID).

Table II Extended Polarization by components: ER(α, ρ*)

N^{er} of groups	ER($\alpha = 1, \rho$*)			ER($\alpha = 1.3, \rho$*)			ER($\alpha = 1.6, \rho$*)		
	2	3	4	2	3	4	2	3	4
US									
1974	0.2444	0.1971	0.1571	0.1998	0.1431	0.1048	0.1639	0.1044	0.0703
1979	0.2346	0.1887	0.1500	0.1914	0.1365	0.0995	0.1565	0.0992	0.0663
1986	0.2535	0.2034	0.1619	0.2072	0.1474	0.1078	0.1698	0.1073	0.0722
1991	0.2557	0.2049	0.1634	0.2092	0.1484	0.1089	0.1717	0.1080	0.0730
1994	0.2730	0.2189	0.1743	0.2238	0.1593	0.1166	0.1842	0.1166	0.0785
1997	0.2753	0.2219	0.1776	0.2267	0.1628	0.1201	0.1877	0.1204	0.0820
2000	0.2707	0.2185	0.1743	0.2228	0.1601	0.1175	0.1843	0.1182	0.0799
UK									
1974	0.2049	0.1639	0.1313	0.1680	0.1194	0.0883	0.1383	0.0875	0.0598
1979	0.1998	0.1594	0.1269	0.1634	0.1157	0.0849	0.1341	0.0844	0.0572
1986	0.2177	0.1738	0.1377	0.1786	0.1266	0.0919	0.1472	0.0927	0.0617
1991	0.2510	0.2009	0.1610	0.2063	0.1472	0.1091	0.1705	0.1087	0.0746
1995	0.2467	0.1978	0.1560	0.2029	0.1451	0.1045	0.1678	0.1073	0.0705
1994	0.2501	0.2010	0.1588	0.2063	0.1481	0.1072	0.1714	0.1102	0.0730
1999	0.2588	0.2077	0.1653	0.2134	0.1527	0.1118	0.1772	0.1132	0.0762
Sweden									
1975	0.1666	0.1325	0.1055	0.1355	0.0955	0.0698	0.1103	0.0690	0.0463
1981	0.1407	0.1128	0.0904	0.1144	0.0811	0.0598	0.0931	0.0584	0.0397
1987	0.1631	0.1314	0.1058	0.1325	0.0945	0.0703	0.1078	0.0682	0.0469
1992	0.1669	0.1348	0.1080	0.1359	0.0973	0.0717	0.1108	0.0704	0.0478
1995	0.1594	0.1307	0.1054	0.1297	0.0943	0.0703	0.1057	0.0684	0.0472
2000	0.1782	0.1438	0.1151	0.1457	0.1044	0.0769	0.1194	0.0762	0.0517
Germany									
1973	0.2024	0.1624	0.1298	0.1659	0.1180	0.0867	0.1365	0.0862	0.0584
1978	0.1957	0.1574	0.1256	0.1604	0.1147	0.0841	0.1321	0.0840	0.0567
1981	0.1863	0.1494	0.1191	0.1522	0.1084	0.0796	0.1246	0.0790	0.0534
1983	0.1894	0.1526	0.1215	0.1555	0.1112	0.0813	0.1281	0.0815	0.0547
1984	0.2051	0.1662	0.1330	0.1678	0.1210	0.0891	0.1377	0.0887	0.0601
1989	0.1900	0.1541	0.1231	0.1554	0.1121	0.0823	0.1275	0.0820	0.0554
1994	0.2001	0.1620	0.1285	0.1639	0.1178	0.0857	0.1348	0.0862	0.0574
2000	0.1930	0.1549	0.1241	0.1580	0.1121	0.0828	0.1298	0.0815	0.0556
Canada									
1975	0.2299	0.1849	0.1472	0.1875	0.1337	0.0978	0.1532	0.0971	0.0652
1981	0.2223	0.1777	0.1417	0.1816	0.1286	0.0943	0.1487	0.0935	0.0631
1987	0.2183	0.1745	0.1387	0.1785	0.1265	0.0924	0.1463	0.0921	0.0618
1991	0.2122	0.1696	0.1349	0.1734	0.1228	0.0896	0.1421	0.0893	0.0599
1994	0.2165	0.1722	0.1371	0.1771	0.1246	0.0911	0.1454	0.0905	0.0609
1997	0.2143	0.1717	0.1364	0.1753	0.1245	0.0908	0.1438	0.0907	0.0607
1998	0.2261	0.1818	0.1445	0.1850	0.1323	0.0965	0.1520	0.0969	0.0648
2000	0.2179	0.1751	0.1403	0.1785	0.1271	0.0939	0.1468	0.0927	0.0633

five countries. These differences are mostly due to ER polarization, but are partly compensated for by the degree of within-group dispersion. Indeed, the groups are more sharply defined in Sweden than in the UK or the US. Germany shows a degree

Table III Extended polarization by components: Intra-group dispersion $G(f) - G(\rho^*)$

N^{er} of groups	2		3		4	
	Absolute level	Relative (% Gini)	Absolute level	Relative (% Gini)	Absolute level	Relative (% Gini)
	$G(f) - G(\rho^*)$	$\frac{G(f)-G(\rho^*)}{G(f)} \bullet 100$	$G(f) - G$ $G(\rho^*)$	$\frac{G(f)-G(\rho^*)}{G(f)} \bullet 100$	$G(f) - G$ $G(\rho^*)$	$\frac{G(f)-G(\rho^*)}{G(f)} \bullet 100$
US						
1974	0.1034	29.7	0.0486	14.0	0.0282	8.1
1979	0.0965	29.1	0.0448	13.5	0.0259	7.8
1986	0.1028	28.8	0.0480	13.5	0.0278	7.8
1991	0.1033	28.8	0.0479	13.3	0.0278	7.7
1994	0.1111	28.9	0.0519	13.5	0.0302	7.9
1997	0.1148	29.4	0.0536	13.7	0.0308	7.9
2000	0.1134	29.5	0.0526	13.7	0.0303	7.9
UK						
1974	0.0822	28.6	0.0388	13.5	0.0223	7.8
1979	0.0771	27.8	0.0363	13.1	0.0216	7.8
1986	0.0901	29.3	0.0443	14.4	0.0259	8.4
1991	0.0985	28.2	0.0465	13.3	0.0274	7.8
1995	0.0996	28.8	0.0475	13.7	0.0278	8.0
1994	0.1018	28.9	0.0484	13.8	0.0287	8.1
1999	0.1072	29.3	0.0515	14.1	0.0302	8.3
Sweden						
1975	0.0665	28.5	0.0328	14.1	0.0193	8.3
1981	0.0604	30.0	0.0294	14.6	0.0172	8.5
1987	0.0728	30.9	0.0359	15.2	0.0209	8.9
1992	0.0744	30.8	0.0361	15.0	0.0212	8.8
1995	0.0759	32.3	0.0362	15.4	0.0209	8.9
2000	0.0789	30.7	0.0384	14.9	0.0224	8.7
Germany						
1973	0.0853	29.7	0.0404	14.1	0.0235	8.2
1978	0.0823	29.6	0.0390	14.0	0.0226	8.1
1981	0.0766	29.1	0.0367	14.0	0.0213	8.1
1983	0.0795	29.6	0.0369	13.7	0.0214	8.0
1984	0.0932	31.2	0.0458	15.4	0.0267	9.0
1989	0.0839	30.6	0.0398	14.5	0.0236	8.6
1994	0.0867	30.2	0.0405	14.1	0.0237	8.3
2000	0.0824	29.9	0.0392	14.2	0.0226	8.2
Canada						
1975	0.0951	29.3	0.0444	13.7	0.0259	8.0
1981	0.0899	28.8	0.0427	13.7	0.0249	8.0
1987	0.0884	28.8	0.0422	13.8	0.0246	8.0
1991	0.0869	29.1	0.0418	14.0	0.0243	8.1
1994	0.0871	28.7	0.0421	13.9	0.0243	8.0
1997	0.0883	29.2	0.0421	13.9	0.0244	8.1
1998	0.0949	29.6	0.0455	14.2	0.0266	8.3
2000	0.0927	29.8	0.0443	14.3	0.0258	8.3

Table IV Intra-group Dispersion by Income groups $G(f_i)$

N^{er} of Groups	2		3			4			
Groups(*)	1	2	1	2	3	1	2	3	4
US									
1974	0.248	0.197	0.245	0.100	0.175	0.258	0.087	0.076	0.166
1979	0.244	0.174	0.239	0.097	0.143	0.245	0.085	0.071	0.129
1986	0.259	0.189	0.249	0.106	0.158	0.251	0.093	0.078	0.145
1991	0.255	0.193	0.242	0.107	0.161	0.241	0.092	0.081	0.146
1994	0.269	0.216	0.257	0.114	0.187	0.261	0.097	0.086	0.178
1997	0.250	0.247	0.230	0.114	0.233	0.224	0.094	0.089	0.233
2000	0.248	0.241	0.228	0.112	0.225	0.221	0.092	0.088	0.221
UK									
1974	0.162	0.182	0.131	0.087	0.168	0.116	0.069	0.067	0.169
1979	0.161	0.160	0.131	0.085	0.136	0.124	0.070	0.066	0.127
1986	0.189	0.190	0.182	0.092	0.171	0.210	0.071	0.074	0.165
1991	0.204	0.217	0.168	0.107	0.199	0.156	0.088	0.082	0.204
1995	0.207	0.217	0.185	0.105	0.197	0.189	0.083	0.083	0.191
1994	0.199	0.238	0.173	0.105	0.227	0.172	0.084	0.087	0.232
1999	0.219	0.241	0.201	0.109	0.233	0.208	0.087	0.087	0.238
Sweden									
1975	0.163	0.117	0.154	0.073	0.098	0.164	0.062	0.049	0.093
1981	0.148	0.103	0.159	0.060	0.084	0.191	0.048	0.045	0.079
1987	0.189	0.121	0.214	0.067	0.108	0.279	0.060	0.049	0.106
1992	0.173	0.136	0.185	0.068	0.122	0.222	0.057	0.053	0.118
1995	0.181	0.135	0.217	0.063	0.125	0.265	0.054	0.051	0.124
2000	0.162	0.164	0.159	0.073	0.159	0.175	0.059	0.059	0.164
Germany									
1973	0.175	0.181	0.161	0.084	0.164	0.161	0.066	0.068	0.158
1978	0.162	0.181	0.143	0.081	0.170	0.137	0.064	0.066	0.169
1981	0.158	0.159	0.140	0.078	0.143	0.138	0.065	0.061	0.137
1983	0.152	0.179	0.129	0.078	0.167	0.118	0.061	0.065	0.163
1984	0.197	0.192	0.205	0.084	0.195	0.243	0.068	0.065	0.205
1989	0.172	0.176	0.161	0.077	0.172	0.165	0.064	0.062	0.178
1994	0.178	0.183	0.167	0.080	0.167	0.170	0.066	0.068	0.161
2000	0.172	0.171	0.162	0.080	0.155	0.165	0.063	0.065	0.149
Canada									
1975	0.237	0.173	0.229	0.095	0.146	0.234	0.083	0.070	0.135
1981	0.211	0.171	0.196	0.094	0.145	0.199	0.079	0.070	0.134
1987	0.199	0.173	0.185	0.092	0.148	0.186	0.076	0.072	0.139
1991	0.196	0.170	0.185	0.090	0.147	0.190	0.073	0.069	0.141
1994	0.196	0.171	0.185	0.093	0.146	0.189	0.073	0.071	0.137
1997	0.197	0.175	0.185	0.090	0.151	0.191	0.074	0.070	0.142
1998	0.209	0.192	0.198	0.094	0.175	0.207	0.079	0.073	0.174
2000	0.200	0.190	0.189	0.090	0.175	0.195	0.074	0.071	0.173

(*) Groups are ordered by (ascending) average income.

of polarization similar to Sweden, while polarization in Canada is higher (though still significantly below that in the UK and the US). We obtain similar results for higher values of α and for two and four groups.

Table V Inter-group cut-off incomes (relative to the mean income), y_i/μ

N^{er} of groups	2	3		4		
Between groups(*)	1 and 2	1 and 2	2 and 3	1 and 2	2 and 3	3 and 4
US						
1974	1	0.728	1.357	0.589	1.004	1.603
1979	1	0.733	1.335	0.593	1.001	1.553
1986	1	0.713	1.370	0.574	1.009	1.626
1991	1	0.707	1.372	0.574	1.009	1.640
1994	1	0.696	1.408	0.553	1.006	1.702
1997	1	0.707	1.428	0.572	1.018	1.772
2000	1	0.710	1.416	0.573	1.010	1.739
UK						
1974	1	0.773	1.310	0.672	1.019	1.545
1979	1	0.770	1.302	0.672	1.019	1.511
1986	1	0.758	1.328	0.635	0.977	1.540
1991	1	0.727	1.400	0.610	1.033	1.703
1995	1	0.735	1.391	0.602	0.990	1.648
1994	1	0.735	1.408	0.610	1.000	1.697
1999	1	0.725	1.410	0.598	1.005	1.728
Sweden						
1975	1	0.805	1.235	0.698	1.010	1.371
1981	1	0.827	1.181	0.726	0.981	1.288
1987	1	0.803	1.212	0.688	0.991	1.341
1992	1	0.805	1.225	0.695	0.990	1.369
1995	1	0.807	1.198	0.701	0.986	1.341
2000	1	0.801	1.255	0.694	0.990	1.426
Germany						
1973	1	0.773	1.292	0.667	0.999	1.516
1978	1	0.785	1.293	0.681	1.003	1.506
1981	1	0.790	1.276	0.685	1.009	1.457
1983	1	0.790	1.281	0.691	0.999	1.489
1984	1	0.777	1.296	0.652	0.991	1.497
1989	1	0.791	1.277	0.684	1.006	1.476
1994	1	0.777	1.292	0.662	0.986	1.489
2000	1	0.779	1.269	0.677	0.996	1.479
Canada						
1975	1	0.737	1.328	0.603	1.008	1.546
1981	1	0.745	1.322	0.625	1.009	1.545
1987	1	0.751	1.321	0.631	1.001	1.535
1991	1	0.758	1.308	0.637	0.994	1.510
1994	1	0.749	1.311	0.633	0.990	1.527
1997	1	0.758	1.313	0.637	0.992	1.523
1998	1	0.751	1.339	0.622	0.998	1.562
2000	1	0.754	1.315	0.642	1.006	1.558

(*) Groups are ordered by (ascending) average income.

The time profile of extended polarization can be visualized in Figure 2. The evolution through the period shows two types of patterns. One type displays increases

Table VI Group population shares, π_i

N^{er} of groups	2		3			4			
Groups(*)	1	2	1	2	3	1	2	3	4
US									
1974	59.19	40.81	39.24	39.06	21.70	28.13	31.32	26.86	13.69
1979	57.48	42.52	37.92	38.74	23.34	27.27	30.28	26.58	15.87
1986	58.95	41.05	39.28	37.90	22.83	28.72	30.74	25.81	14.73
1991	59.63	40.37	39.31	37.86	22.83	29.39	30.77	25.39	14.46
1994	60.82	39.18	40.89	38.01	21.10	30.15	31.02	25.50	13.33
1997	63.19	36.81	42.90	38.83	18.26	32.17	32.13	25.13	10.58
2000	62.86	37.14	42.32	39.10	18.58	31.44	31.96	25.41	11.18
UK			2.28						
1974	60.91	39.09	41.37	37.60	21.03	31.56	30.84	25.06	12.53
1979	59.55	40.45	41.11	35.66	23.23	31.46	29.45	24.77	14.31
1986	61.28	38.72	41.60	37.39	21.01	26.73	32.96	26.60	13.71
1991	62.33	37.67	43.84	36.40	19.76	34.13	30.26	24.18	11.43
1995	62.66	37.34	43.76	37.16	19.08	30.20	31.80	25.61	12.40
1994	64.18	35.82	45.10	37.15	17.76	32.14	32.03	25.01	10.82
1999	64.02	35.98	44.03	37.92	18.05	31.78	32.62	25.01	10.59
Sweden			2.44						
1975	54.11	45.89	36.58	37.04	26.37	25.73	29.31	27.38	17.58
1981	53.58	46.42	32.54	39.52	27.94	20.19	31.11	28.46	20.23
1987	53.03	46.97	32.82	39.89	27.29	20.36	31.85	29.92	17.86
1992	55.73	44.27	34.94	40.33	24.74	22.77	32.06	28.69	16.48
1995	54.98	45.02	31.14	42.94	25.92	19.85	33.43	30.06	16.66
2000	59.11	40.89	38.57	39.79	21.64	25.08	33.10	28.29	13.52
Germany			1.78						
1973	60.79	39.21	39.54	39.16	21.30	28.48	32.29	26.00	13.23
1978	60.96	39.04	40.75	38.74	20.51	29.19	32.09	26.15	12.57
1981	58.39	41.61	39.93	37.62	22.45	28.56	30.80	27.06	13.58
1983	61.43	38.57	40.60	39.08	20.33	29.41	31.88	25.82	12.89
1984	59.60	40.40	38.98	40.73	20.29	25.79	33.11	28.50	12.60
1989	59.57	40.43	39.10	40.10	20.80	28.21	31.95	27.03	12.81
1994	60.36	39.64	39.54	39.68	20.78	26.93	32.21	27.18	13.68
2000	59.89	40.11	37.86	39.73	22.41	27.01	32.67	26.67	13.65
Canada			1.69						
1975	57.27	42.73	37.91	38.54	23.55	27.83	30.02	26.70	15.46
1981	58.53	41.47	38.99	37.62	23.39	28.44	30.79	26.01	14.77
1987	59.20	40.80	39.80	37.34	22.85	27.97	31.25	26.24	14.55
1991	58.84	41.16	39.16	37.78	23.06	27.10	31.36	26.49	15.05
1994	59.59	40.41	39.22	37.27	23.50	27.13	31.83	25.83	15.21
1997	59.42	40.58	39.44	38.18	22.38	27.22	31.75	26.36	14.67
1998	59.88	40.12	40.58	38.33	21.08	28.05	31.73	26.79	13.43
2000	60.49	39.51	39.37	38.96	21.67	28.56	32.35	26.25	12.84

(*) Groups are ordered by (ascending) average income.

in simple polarization that are partly offset by increases in the spread within the groups. In the net, we record increases in extended polarization. This is the case for

Table VII Mean incomes by groups (relative to the mean), μ_i/μ

N^{er} of groups	2		3			4			
group (*)	1	2	1	2	3	1	2	3	4
US									
1974	0.587	1.599	0.448	1.009	1.983	0.365	0.790	1.255	2.284
1979	0.592	1.552	0.451	1.009	1.876	0.369	0.794	1.237	2.080
1986	0.570	1.618	0.427	1.008	1.972	0.348	0.785	1.276	2.237
1991	0.571	1.633	0.427	0.998	1.990	0.354	0.786	1.281	2.276
1994	0.551	1.697	0.410	1.004	2.136	0.333	0.768	1.296	2.481
1997	0.564	1.748	0.431	1.011	2.313	0.362	0.783	1.319	2.845
2000	0.569	1.729	0.433	1.009	2.272	0.362	0.781	1.298	2.744
UK									
1974	0.664	1.524	0.561	1.007	1.850	0.511	0.837	1.244	2.146
1979	0.665	1.494	0.566	1.006	1.758	0.520	0.835	1.239	1.982
1986	0.645	1.562	0.538	1.002	1.911	0.451	0.786	1.215	2.170
1991	0.597	1.666	0.488	1.016	2.107	0.437	0.807	1.319	2.518
1995	0.606	1.661	0.497	1.016	2.123	0.421	0.774	1.260	2.453
1994	0.610	1.698	0.504	1.015	2.227	0.438	0.783	1.278	2.670
1999	0.596	1.719	0.478	1.011	2.249	0.409	0.782	1.296	2.744
Sweden									
1975	0.692	1.363	0.592	1.018	1.541	0.525	0.849	1.184	1.661
1981	0.737	1.303	0.625	0.994	1.445	0.531	0.853	1.121	1.525
1987	0.693	1.347	0.564	1.001	1.523	0.452	0.839	1.154	1.655
1992	0.700	1.377	0.582	0.999	1.592	0.490	0.841	1.156	1.741
1995	0.710	1.354	0.562	0.985	1.550	0.451	0.849	1.137	1.710
2000	0.699	1.436	0.593	1.003	1.720	0.510	0.833	1.175	1.951
Germany									
1973	0.667	1.516	0.552	0.996	1.838	0.487	0.826	1.214	2.108
1978	0.679	1.501	0.575	1.005	1.834	0.513	0.833	1.212	2.116
1981	0.681	1.448	0.584	1.010	1.723	0.523	0.837	1.206	1.961
1983	0.692	1.491	0.589	1.001	1.820	0.531	0.838	1.197	2.076
1984	0.656	1.508	0.537	1.007	1.876	0.446	0.812	1.198	2.179
1989	0.681	1.470	0.569	1.008	1.794	0.505	0.843	1.200	2.061
1994	0.668	1.505	0.554	1.000	1.848	0.476	0.817	1.188	2.090
2000	0.678	1.481	0.559	0.989	1.765	0.491	0.830	1.201	2.022
Canada									
1975	0.599	1.538	0.462	1.008	1.853	0.387	0.803	1.239	2.074
1981	0.620	1.536	0.496	1.004	1.833	0.426	0.808	1.247	2.070
1987	0.631	1.535	0.515	1.004	1.839	0.440	0.803	1.236	2.075
1991	0.639	1.516	0.522	1.002	1.809	0.444	0.803	1.220	2.024
1994	0.637	1.536	0.517	0.993	1.817	0.439	0.798	1.227	2.038
1997	0.639	1.528	0.521	1.003	1.840	0.442	0.803	1.220	2.065
1998	0.622	1.564	0.504	1.012	1.932	0.423	0.797	1.237	2.211
2000	0.640	1.552	0.514	0.997	1.887	0.445	0.817	1.240	2.207

(*) Groups are ordered by (ascending) average income.

the UK and the US, and also for Sweden (except for a minor decrease in two-group polarization). All these countries have experienced a process of increased within-group inequality, with less sharply defined groups by the end of the period.

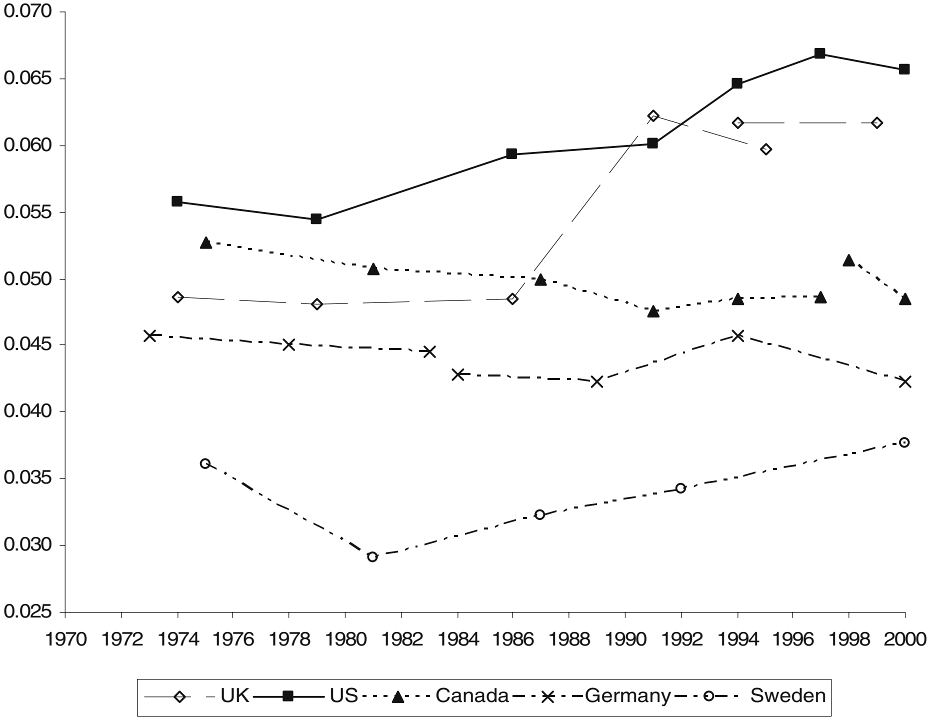

Figure 2 Polarization in five OECD countries: three groups and $\alpha = 1.6$.

At the same time, as Table IV reveals, the three countries differ in the evolution of within-group inequality by income groups. In the three-group case, Sweden has recorded an increase in spread in the upper income class only, where within-group inequality has increased by 65%. The US has experienced a decrease in the dispersion within the poor class and increases of 10% in the middle class and of 30% within the upper class. As for the UK, we observe an increase in spread within each of the three income classes: 55% within the poor, 25% within the middle class, and 40% among the upper class.

The second pattern consists of a decrease in simple polarization, combined with stability or moderate decline in within-group dispersion. The overall effect is a decrease in extended polarization. This is the case of Canada and Germany. In Canada (Table IV) we observe a fall in the dispersion within the poor and middle classes, together with a 25% increase in the dispersion within the upper class. For Germany, the only significant change is a moderate reduction in dispersion within the upper class.

At the beginning of the period only Sweden stood out as a country with distinctly low polarization. Germany was as polarized as the UK, and both were nearly as polarized as Canada and the US. By the end of the period two types of countries emerge with clearly different levels of polarization: US and UK are the two highly polarized countries, while Sweden, Germany and Canada all exhibit low polarization, with a significant gap in between. The distribution of countries by level of polarization has itself become more polarized!

5. A final remark

We conclude with a remark on the number of groups to be used. In Section 3 we have intentionally left this issue to the taste of the analyst. We do not have sufficiently strong arguments in favor of any particular solution. At the same time, we realize that the assumption of a fixed number of groups is not entirely satisfactory, though this is to be distinguished from the additional note of caution sounded at the end of Section 3.2.

A possible point of view that deserves some attention is the following. Since the object of the exercise is to capture the degree of polarization, one might choose the number of groups to provide the sharpest view of polarization; that is, choose the number of groups to maximize extended polarization. For instance, when $\alpha = 1$, extended polarization is generally maximized when society is divided into three groups: Poor, middle and rich. In fact, this is true for the five countries in our study. For higher values of α the two-group representation turns out to yield slightly higher levels of extended polarization. The endogenous choice of groups represents an interesting avenue of research that we do not pursue here.

Acknowledgements Esteban and Gradín acknowledge the financial support of Fundación Barrié de la Maza. Esteban is a member of Barcelona Economics, funded by the Generalitat de Catalunya, and gratefully acknowledges financial support from the Spanish Ministerio de Educación y Ciencia, grant SEC-2003-1961, the Instituto de Estudios Fiscales, and from the European Commission, contract CIT2-CT-2004-506084. Gradín acknowledges financial support from the Spanish Ministerio de Educación y Ciencia, grant SEJ2004-07373-C03-02. Ray acknowledges financial support under National Science Foundation Grant 0241070. We are grateful to R. Roy and three anonymous referees for their useful comments.

References

1. Anderson, G.: Toward an empirical analysis of polarization. J. Econ. **122**, 1–26 (2004)
2. Anderson, G.: “Polarization”, mimeo, Department of Economics, University of Toronto (2005)
3. Aghevli, B.B., Mehran, F.: Optimal grouping of income distribution data. J. Am. Stat. Assoc. **76**, 22–26 (1981)
4. Chakravarty, S., Majumder, A.: Inequality, polarization and welfare: Theory and applications. Aust. Econ. Papers **40**, 1–13 (2001)
5. Chakravarty, S., Majumder, A., Roy, S.: A treatment of absolute indices of polarization. Mimeo, forthcoming in The Japanese Economic Review (2002)
6. Davies, J.B., Shorrocks, A.F.: Optimal grouping of income and wealth data. J. Econ. **42**, 97–108 (1989)
7. Duclos, J-Y., Esteban, J., Ray, D.: Polarization: Concepts, measurement, estimation. Econometrica **72**, 1737–1772 (2004), reprinted in Christopher Barrett (ed.) The Social Economics of Poverty: On Identities, Groups, Communities, and Networks. Routledge, London (2005)
8. Esteban, J., Ray, D.: On the measurement of polarization. Econometrica **62**, 819–851 (1994)
9. Esteban, J., Gradín, C., Ray, D.: Extensions of a Measure of Polarization, with an Application to the Income Distributions of Five OECD Countries. Working Paper 24, Instituto de Estudios Económicos de Galicia-P. Barrié de la Maza, A Coruña, Spain. Also in: Luxembourg Income Study (LIS) Working Paper Series, 218, Maxwell School of Citizenship and Public Affairs, Syracuse University, Syracuse, New York (1999)
10. Gradín, C.: Polarization by sub-populations in Spain, 1973–91. Rev. Income Wealth **46**(4), 457–474 (2000)
11. Love, R., Wolfson, M.C.: Income Inequality: Statistical Methodology and Canadian Illustrations, Catalogue 13–559 Occasional. Ottawa, Statistics Canada (1976)
12. Montalvo, J., Reynal-Querol, M.: Ethnic polarization, potential conflict, and civil wars. Am. Econ. Rev. **95**, 796–816 (2005)

13. Reynal-Querol, M.: Ethnicity, political systems, and civil wars. J. Confl. Resolut. **46**, 29–54 (2002)
14. Salas, R., Rodríguez, J.G.: Extended bi-polarization and inequality measures. Res. Econ. Inequal. **9**, 69–83 (2002)
15. Wang, Y., Tsui, K.: Polarization orderings and new classes of polarization indices. J. Public Econ. Theory **2**(3), 349–363 (2000)
16. Wolfson, M.C.: When inequalities diverge. Am. Econ. Rev. **84**, Papers and Proceedings, 353–358 (1994)
17. Zhang, X., Kanbur, R.: What difference do polarization measures make? An application to China. J. Dev. Stud. **37**(3), 85–98 (2001)