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Conflict and Distribution*

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We develop a behavioral model that links the level and pattern of social conflict to the societywide distribution of individual characteristics. The model can be applied to groups that differ in characteristics such as wealth, ethnicity, religion, and political ideology. We settle questions of existence and uniqueness of conflict equilibrium. Conflict is seen to be closely connected with the bimodality of the underlying distribution of characteristics. However, in general, the conflict–distribution relationship is nonlinear and surprisingly complex. Our results on conflict patterns also throw light on the phenomena of extremism and moderation. *Journal of Economic Literature* Classification Numbers: D63, D72, D74. © 1999 Academic Press

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1. INTRODUCTION

The aim of this paper is to develop a behavioral model that links social "conflict" to the society-wide distribution of individual characteristics. We study the changes in the equilibrium *level* and (inter-group) *pattern* of conflict generated by shifts in the distribution of individuals over the set of

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groups that favour different outcomes. Our question is, What sort of distributions are likely to be most highly correlated with conflict?

We conceive of conflict as a situation in which, in the absence of a collective decision rule, social groups with opposed interests incur losses in order to increase the likelihood of obtaining their preferred outcomes. This analytical core is fairly standard and simple and has been used in several contexts.[1] In contrast with most of the literature, however, we explicitly allow for the possibility that groups may derive utility from the preferred outcomes of *other* groups. This imparts, in many situations, a natural "metric" across groups, two groups being close if they value the preferred outcomes of one another in terms similar to their own.

For tractability, we assume that all individuals preferring the same outcome also coincide in their valuations of other outcomes. Therefore, there are as many preference orderings as the number of available outcomes. Individuals with the same ordering form an "interest group" and act in a coordinated fashion.[2] As constructed, the model can be applied to groups that differ not only in their levels of income and wealth, but also in other characteristics, such as ethnicity, religion, and political ideology.

The particular outcome is seen as the realization of a lottery, with outcome probabilities related to the share of resources expended by each group. Conflict is viewed simply as the equilibrium sum of resources that are dissipated in the struggle for preferred outcomes.

With this abstract representation of a conflictual situation, we address the questions that motivate our work. As already stated, our general concern is the study of the relationship between equilibrium conflict and the shape of the frequency distribution of agents over the given set of alternative preferences.

Preliminary questions arise. We settle the question of existence of a conflict equilibrium, and provide what we believe to be a general and new condition for the uniqueness of equilibrium (counterexamples to uniqueness are provided in the absence of this condition). See Section 3. In what follows, we take it that uniqueness obtains, so that we can legitimately assign to each parametric situation a unique level and pattern of conflict.

[1] Among the several contributions, see [1, 8–11, 23–25] (see also the recent edited volume by Garfinkel and Skaperdas [7]). There are also a number of particular cases of conflict of interests that have been the object of specific analyses, stemming from the rent-seeking problem as formulated in [27]. It is beyond the scope of this paper to list the enormous number of contributions to the rent-seeking literature; many of these were published in the journal *Public Choice*. The contributions include models of patent races, or of influence activities within an organization (see [17, 18]). Our modeling of conflict is a direct heir of these previous works. There is also a recent literature that examines the connections between social conflict and economic growth; see, e.g., [3, 14, 26]. For a survey, see [2].

[2] In particular, we do not discuss here the important free-rider problem that is internal to each group—more on this below.

Section 4 begins our study of the relationship between conflict and distribution. Our first observation, which is completely intuitive, is that an increase in the "utility distance" between any pair of groups leads, ceterius paribus, to an increase in societal conflict (Proposition 4.1). Our second observation (Proposition 4.2) states that conflict is always maximized at some symmetric *bimodal* distribution of the population, provided that cross-group preferences satisfy a symmetry condition. Thus the emergence of a "twin-peaks property" in distribution is related closely to rising conflict.[3] It should be noted that this property is often at odds with the Pigou–Dalton principle that underlies inequality measurement, but is closely related to the notion of *polarization* [5, 28].[4]

Further probing of the conflict-distribution relationship yields surprisingly complicated findings, even in special cases. We focus first on the case of pure contests: a situation in which each group dislikes equally the outcomes of all other groups. It turns out (Propositions 4.3 and 4.4) that the uniform distribution of population over three groups is always a *local* maximizer of conflict but never a *global* maximizer (that distinction belonging, as already observed, to the bimodal distribution). The same may be true (though not always) for uniform distributions over a larger number of groups. This reveals, in particular, the highly nonlinear relationship between conflict and distribution even in the special case of contests. One way to see this is to conduct the thought experiment of moving population mass, starting from the symmetric distribution over three groups and ending at the symmetric distribution over two groups, and applying the results stated above.

In Section 5, we take up the question, What is the *pattern* of conflict over groups as a function of their population weights and their position in the general structure of preferences? This question is difficult and we do not have general answers. For contests matters are simple: it turns out that larger groups are always more contentious (per capita) than their smaller counterparts. In the more general case, however, population size cannot be the only determinant: the "spatial" structure of preferences also counts. We turn here to the analysis of another special case in which groups may be visualized as being arranged on a "line." This case yields interesting insight into what might be called the phenomenon of *extremism*: with a uniform distribution of population over groups, it turns out that the groups on the "sides" of the line are more conflictual: they put in a larger per capita share

[3] The twin-peaks phenomenon (in the terminology of Cowell) was particularly noticeable in the United Kingdom during the 1980s. In the United States, Morris *et al.* [19] find signs of polarization in the wage distribution. Quah [22] studies the twin-peaks property for income distributions *across* countries. On related matters see [4, 12, 13].

[4] In an extended version of the current paper [6] we discuss the relationship between conflict and polarization in detail.

of resources. But more can be said as we study the interplay between population distribution and preference structure: if these extremist groups shrink in population size, there comes a point where they switch from contributing more than the average level of conflict to less than the average level: the situation changes from extremist to what might be called "moderate" (Proposition 5.3). This uncovers an interesting connection betweem the distribution of population over various groups and the prevalence of extremism. If the distribution of population is spread out to start with, *observable* behavior appears to be more extremist than the *true* distribution of opinion. This might be in line with patterns of behavior in countries such as India, where the expression of interreligious conflict might appear more extremist than the true distribution of views, the relatively large moderate view failing to capture the headlines. On the other hand, more homogeneous societies might *appear* to be more consensual than they really are: this is the upshot of the second result (on moderation) and may apply to, say, the Scandinavian countries. A corollary of this is that the expression of views swings more widely than the underlying distribution of preferences across outcomes, which might explain why in some circumstances, small changes in distribution may be give rise to relatively large changes in observed patterns of social conflict.

Section 6 briefly describes some connections between the current work and the measure of polarization developed in [5]. All proofs are relegated to Section 7.

2. A MODEL OF CONFLICT

Consider a society composed of a unit measure of individuals, situated in G groups. Let n_i be the number of individuals in group i, so that $\sum_{i=1}^{G} n_i = 1$.

Society must choose one issue or outcome over G possible issues, and we identify issue i as the outcome most preferred by group i. We will allow for arbitrary preferences by members of group i over the other issues, but take it that they strictly prefer their own outcome to any of these others.

In this paper, each outcome will be thought of as a pure public good for the group members, so that per-capita utilities over the outcome are well-defined irrespective of the membership distribution of the groups. Thus define u_{ij} to be the utility derived by a member of group i if issue j is chosen by society. By the assumption in the previous paragraph, $u_{ii} > u_{ij}$ for all i, j with $i \neq j$.

It is obvious that one cannot describe how rational individuals will behave under a potentially conflictual situation without first specifying a mechanism that resolves conflict by making a choice over the space of issues. We assume here that agents can (probabilistically) influence the outcome of the decision process by allocating resources into lobbying activities. Specifically, we think of the decision process as a lottery, with the probability of each of G possible outcomes depending on the vector of resources provided by the G groups.

Denote by r_{ij} the resources expended by a typical individual of group i in support of outcome j, so that the total amount spent by group i is $n_i \sum_{j=1}^{G} r_{ij} = n_i r_i$, where r_i is the total amount of resources expended per group member. Then the total resources devoted to lobbying by society is $R \equiv \sum_{i=1}^{G} n_i r_i$. We will use R as a measure of societal *conflict*.

Resources are acquired at a cost to each individual. We denote by $c(r)$ the individual cost of supplying r. Assume that

ASSUMPTION 1. c is continuous, increasing, and thrice continuously differentiable, with $c(0)=0$, $c'(r)>0$, and $c''(r)>0$ for all $r>0$, and with $c'(0) \equiv \lim_{r \downarrow 0} c'(r) = 0$.

Let p_j be the probability that issue j will be chosen. Then the expected utility of a member of group i who expends resources r_i is given by

$$\sum_{j=1}^{G} p_j u_{ij} - c(r_i). \tag{1}$$

To determine the probabilities, we suppose that

$$p_j = s_j \equiv \frac{\sum_{i=1}^{G} n_i r_{ij}}{\sum_{k=1}^{G} n_k r_k} = \frac{\sum_{i=1}^{G} n_i r_{ij}}{R} \tag{2}$$

for all $j = 1, ..., G$, provided that $R>0$.[5] Thus the probability that group i will win the lottery is taken to be exactly equal to the share of total resources expended in support of alternative i.

To complete the specification of the model, we need to describe the outcome when $R=0$. We take this to be an arbitrary probability vector $\{\bar{p}_1, ..., \bar{p}_G\}$.[6]

This model is quite general. It covers, for instance, the case of *pure contests* that has received extensive attention in the literature. This case is characterized by group preferences that *only* place value on the most preferred issue: $u_{ij}=0$ for all i, j, with $i \neq j$, and $u_{ii}=1$ for all i.[7]

[5] We could just as well write $w_i = c(r_i)$, so that $p_i = n_i c^{-1}(w_i) / \sum_{k=1}^{G} n_k c^{-1}(w_i)$. This expression is related to the class of "conflict technologies" proposed by Hirshleifer [11] and axiomatized in [24].

[6] There is, of course, no way to complete the specification of the model at $R=0$ while maintaining continuity of payoffs for all groups. So the game thus defined must have discontinuous payoffs. Fortunately, this does not create existence problems, as we show in Proposition 3.2 below.

[7] To focus on group distribution, we deliberately neglect other asymmetries in the contest case. These are easily incorporated, however.

Contests describe a situation where there is, in effect, no "metric" over the different groups. Alternatively, each group might have natural "positions" that allow us to state if one group is close to or far away from another. For instance, think of the issues (and groups) being arranged on a political spectrum from right to left, or being specific to different income groups but having repercussions for nearby groups, or simply arranged by geography (e.g., the location of a facility such as a public university). A simple case of a "metric" is induced by a *line*, which we may define as the following submodel: There is an ordering of the groups (which we may call 1, 2, ..., G without loss of generality) such that for all i and j with $i < j$, $u_{ij} \geqslant u_{i, j+1}$ and $u_{ij} \leqslant u_{i+1, j}$.

We will return to these specifications below.

Our behavioral framework will embody two important assumptions that are maintained through the paper. First, we assume that no group expends resources on outcomes other than its preferred position. Second, we ignore free-rider problems within each group.

Are these assumptions satisfactory? Not entirely. It may well be that a group decides to support the lobbying activities of some other group. A satisfactory treatment of this issue will have to depart from the present model in one of two ways. One route is to look at "nonconvex" lobbying technologies in which some threshold resource expenditure is needed to influence the success probability at all.[8] [With convex technologies, as assumed in (2) below, groups would always lobby for their own best outcome, so there is no internal inconsistency in the model as it stands.] A second route is to build in an additional stage to the game in which groups first precommit to positions and subsequently lobby for their committed positions. In this case, some groups may lobby for "less radical" positions simply to dilute the intensity of the opposition. We feel that each extension would be interesting in its own right.

Our behavioral framework ignores free-rider problems within each group. The free-rider problem is classical, one that lies at the heart of the Olson thesis (see [20]) on the lobbying efficacy of small groups.[9] However, there is little that we wish to add to this problem and so we assume that external effects within a group are fully internalized by group members.[10]

[8] We are indebted to an anonymous referee for pointing this out.

[9] One might more accurately dub this the Pareto–Olson thesis, for Vilfredo Pareto [21], p. 329] points to exactly the same argument.

[10] It is not true, however, that allowance for free-rider effects will automatically resurrect the Pareto–Olson argument,beacuse in this model we deal with outcomes that are public goods. The dissipative power of large groups is thereby attenuated.

3. BASIC PROPERTIES

3.1. *Existence*

With the closing comments of the previous section in mind, the choice problem faced by group i is easy to describe: given the vector of resources expended by all other groups, choose r_i to maximize (1), subject to (2). This problem is well-defined provided that at least one other group expends a positive quantity of resources, and its solution is completely described by an interior first-order condition. We record this simple observation as

PROPOSITION 3.1. *Fix some group i. Suppose that Assumption* 1 *holds and that $r_j > 0$ for some $j \neq i$. Define $v_{kj} \equiv u_{kk} - u_{kj}$ for all k and j. Then the amount spent by group i is strictly positive, and is completely described by the condition*

$$\sum_{j=1}^{G} s_i s_j v_{ij} = c'(r_i) r_i \tag{3}$$

This first-order condition foreshadows certain features of equilibrium conflict, even before we "close" the model. Specifically, conflict (and the shares of the various groups) will depend, among other things, on the distribution of population over the different groups. Understanding this relationship between conflict and distribution forms the subject matter of the paper. Notice from (3) that *a priori*, the number of individuals in a group has ambiguous implications for conflict. Members of a larger group generate larger externalities for one another, and as such may be cajoled, inspired, or compelled by group leaders to put in more resources *per capita*. This effect is captured (only partly, for the variable in question is endogenous) by the share term s_i: it is associated with a higher value of r_i. On the other hand, for a given *total* population, larger numbers in one group mean smaller numbers elsewhere. This is related (again imperfectly) to smaller values of s_j, for $j \neq i$. With a weaker opposition, the incentive to put in lobbying resources comes down, and (3) captures this as well.[11] The net effects can take either sign. Just how complicated this apparently simple model can become will become evident as we develop the analysis in the sections ahead.

Say that a vector of resources $(r_1^*, ..., r_G^*)$ is an *equilibrium* if for every i the problem (1), subject to (2), is well-defined, and r_i solves this problem. Equilibrium conflict is then given by the quantity $R^* = \sum_{i=1}^{G} r_i^* n_i$. The equilibrium resource shares are $s_i^* = r_i^* n_i / R^*$.

[11] Note that this is quite different from the free-rider issue stressed in [20], which we neglect here.

It is immediate from Proposition 3.1 that an equilibrium must involve strictly positive contributions by every group. The existence of an equilibrium is guaranteed in

PROPOSITION 3.2. *Suppose that Assumption* 1 *holds. Then an equilibrium exists.*

3.2. *Uniqueness*

Before moving on to the analysis of specific cases, it is of interest to inquire whether the model, as it stands, is enough to yield a unique equilibrium. The following proposition provides sufficient conditions that guarantee uniqueness.

PROPOSITION 3.3. *Suppose that* $c'''(r) \geqslant 0$ *for all* r. *Then there is a unique equilibrium.*

The condition in the proposition does have bite, as the following example demonstrates.

EXAMPLE 1. $G=4$, and $n_i = 1/4$ for all i. The values $\{v_{ij}\}$ are described as follows: $v_{ii}=0$ for all i, and for $i \neq j$, $v_{ij}=a$ if $|j-i|$ is odd, and $v_{ij}=b$ if $|j-i|$ is even. Suppose that the cost function c has constant elasticity α, so that it is of the form $c(r)=\alpha^{-1}r^{\alpha}$. By the assumption of convex costs, α must exceed 1. This model is perfectly symmetric, as the schematic depiction in Fig. 1 reveals.

Consider possible equilibrium shares of the form $s_1^* = s_3^* = s \in (0, 1/2)$, and $s_2^* = s_4^* = \frac{1}{2} - s$. Using the first order conditions described by (3) for groups 1 and 3, we see that

$$bs + 2a(\tfrac{1}{2} - s) = s^{\alpha - 1}\lambda, \tag{4}$$

where $\lambda \equiv (R/n)^{\alpha}$. Likewise, for groups 2 and 4,

$$b(\tfrac{1}{2} - s) + 2as = (\tfrac{1}{2} - s)^{\alpha - 1}\, \lambda. \tag{5}$$

One solution is, of course, the symmetric one where $s = \frac{1}{2} - s = \frac{1}{4}$. In that case, (4) or (5) yields an accompanying value of λ equal to $4^{\alpha-2}(b+2a)$.

Now let us explore the possibility of additional solutions to (4) and (5). To do so, define for each $s \in (0, \frac{1}{2})$,

$$\psi(s) \equiv s^{1-\alpha}\{bs + 2a(\tfrac{1}{2} - s)\}.$$

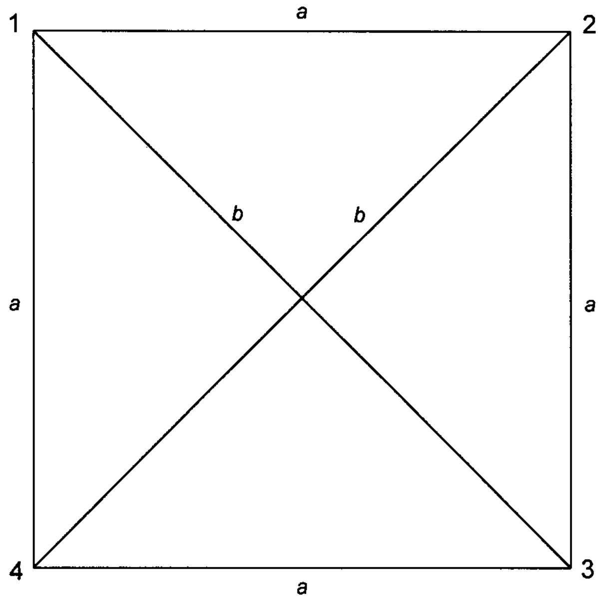

FIG. 1. Diagram to illustrate Example 1.

Obviously, $\psi(s)$ solves for λ as a function of s in (4). Noting the form of (5), it should be obvious that all solutions in $[0, 1]$ to the equation

$$\psi(s) = \psi(\tfrac{1}{2} - s) \tag{6}$$

correspond to equilibria in which groups 1 and 3 put in a common share s, while groups 2 and 4 put in $1 - s$.

When do these additional solutions exist? Note that $\lim_{s \downarrow 0} \psi(s) = \infty$ (because $\alpha > 1$), $\lim_{s \uparrow 1/2} \psi(s) < \infty$, and that ψ is differentiable on $(0, \frac{1}{2})$. It follows right away that multiple solutions exist if $\psi'(\frac{1}{4}) > 0$.

Checking this condition is a matter of simple differentiation. We see that

$$\psi'(\tfrac{1}{4}) = (\tfrac{1}{4})^{1-\alpha} \{(1-\alpha)(b+2a) + (b-2a)\}.$$

It is easy to see that the required condition can always be made to hold provided $\alpha \in (1, 2)$. Simply choose b and a such that

$$\frac{b-2a}{b+2a} > \alpha - 1.$$

Thus for parameters satisfying this condition, there are multiple equilibria.

Notice that uniqueness obtains when the (constant) elasticity of cost is at least 2, but not otherwise. It is easy to check that this requirement is equivalent to the condition that $c'''(r) \geqslant 0$ in the class of all constant-elasticity cost functions.[12]

Before leaving the section on uniqueness, it is worth noting that for special classes of preferences, uniqueness can be achieved without any additional restrictions on the cost function. A leading example is the case of pure contests, in which the equilibrium is always unique (the proof is omitted).

3.3. *Iso-Elastic Cost Functions*

As an illustration of equilibrium, consider briefly the case of isoelastic cost functions: say $c(r) = \alpha^{-1} r^{\alpha}$ for some $\alpha > 1$. In that case, the first-order condition (3) can be rewritten as

$$\sum_{j=1}^{G} s_i s_j v_{ij} = r_i^{\alpha} = s_i^{\alpha} \left(\frac{R}{n_i}\right)^{\alpha}$$

for each i. If we denote by W the $G \times G$ matrix with generic element $w_{ij} \equiv n_i^{\alpha} v_{ij}$, by $\mathbf{s}^{\beta}$ the vector $(s_1^{\beta}, ..., s_G^{\beta})$, and write $\lambda \equiv R^{\alpha}$, then the condition above can be written in matrix form as

$$W\mathbf{s} = \lambda \mathbf{s}^{\alpha - 1}. \tag{7}$$

We know from Example 1 that the solution to this system is generally not unique, but we also know from Proposition 3.3 that there *is* a unique solution provided $\alpha \geqslant 2$. For the case of $\alpha = 2$ this uniqueness result reduces to a Frobenius theorem, as is apparent from (7). The equilibrium resource shares are the entries in the positive (and in the unit simplex, unique) eigenvector of the matrix

$$W = \begin{pmatrix} n_1^2 v_{11} & n_1^2 v_{12} & \cdots & n_1^2 v_{1G} \\ n_2^2 v_{21} & n_2^2 v_{22} & \cdots & n_2^2 v_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ n_G^2 v_{G1} & n_G^2 v_{G2} & \cdots & n_G^2 v_{GG} \end{pmatrix},$$

while R^2 is the (unique) positive eigenvalue of this matrix.

[12] Actually, the proof of Proposition 4 makes it clear that only a weaker condition—that $c'(r)/r$ be nondecreasing in r—is really needed, but we use the slightly stronger version which will help in obtaining other results later in the paper.

4. CONFLICT AND DISTRIBUTION: LEVELS

4.1. *Introduction*

The main purpose of this paper is to pursue a detailed study of the relationship between conflict and the distribution of population across different groups. We divide this exercise into two main parts. In this section, we study the relationship between distributional characteristics and the *level* of conflict in a society. In Section 5, we address the relationship between population distribution and the *pattern* of conflict across different groups.

Quite independently of these issues, there are two features in any model of conflict that need to be highlighted. First, there is the feature of group *size*, and the distribution of these sizes over the different groups in society. This feature is best studied in a world of pure contests, where each group loses equally from the success of any other group. Second, there is the feature of group distances. Contests are inadequate in capturing variations in these. There might be natural metrics over group positions that induce groups to behave differently, even if they are all of similar size.

In the analysis that follows, we study the level and pattern of conflict by paying particular attention to these two features of population distribution.

4.2. *Two General Observations*

We begin with two general observations, and then go on to a more detailed analysis of contests. Consider, first, a situation in which some group distances are increased, while for no two pair of groups is the group distance decreased. Then conflict must increase. Formally

PROPOSITION 4.1. *Assume that $c'''(r) \geqslant 0$ for all $r \geqslant 0$. Consider two societies that are identical in all respects except intergroup distances, which are related as follows: $v'_{ij} \geqslant v_{ij}$ for all groups i and j, with strict inequality for some i and j. Then $R' > R$.*

We turn next to the question of population distribution over a given set of groups and group distances. The central observation here is that of *bimodality*: conflict has a tendency to increase when there are two similar-sized, opposed groups in society. This result can be precisely established under the assumption that inter-group antagonism is symmetric: that is, assuming that $v_{ij} = v_{ji}$.

To begin the discussion of bimodality, let us first consider the case of two groups: $G = 2$. We may write the corresponding first order conditions as

$$n_i n_j r_j v_{ij} = c'(r_i) R^2$$

for $i, j = 1, 2$ and $i \neq j$. This implies right away that

$$\frac{c'(r_1)r_1}{c'(r_2)r_2} = \frac{v_{12}}{v_{21}}.$$

Loosely speaking, this equation tells us that the amount of individual effort contributed by one group *relative* to the other depends on v_{12}/v_{21} and not on the sizes of the respective groups. But this will not be true of the absolute *magnitude* of the efforts, as we shall soon see.

In particular, if one imposes the symmetry condition that $v_{12} = v_{21} \equiv v$, both groups will contribute the same level of per capita effort. Thus $r_1 = r_2 = R$, so that equilibrium conflict R is given by the condition

$$Rc'(R) = vn_1 n_2.$$

It is immediate that R is maximal for $n_1 = n_2 = 1/2$. We have thus shown that for a given two-point support, the symmetric bimodal distribution maximizes conflict over the set of all bimodal distributions with that support.

We now show that under the symmetry condition $v_{ij} = v_{ji}$, equilibrium conflict satisfies the bimodality property and so behaves in this respect as polarization does.

PROPOSITION 4.2. *Suppose that $c'''(r) \geqslant 0$ and that $v_{ij} = v_{ji}$ for all i and j. Then there exists a symmetric bimodal distribution which yields at least as much conflict as any other distribution, and strictly more conflict than any other distribution which is either not symmetric or not bimodal.*

Note that apart from making the symmetry and uniqueness assumptions, we impose no conditions at all on the structure of preferences. In particular, inter-group alienation (as measured by the collection $\{v_{ij}\}$ need not have a linear structure. Thus our behavioral model extends and (conceptually) generalizes Theorem 2 of Esteban and Ray [5], which proves the analogous result for polarization, a concept developed for a "linear structure" of preferences.

At the same time, it should be noted that the symmetry of preferences is crucial to the conflict-maximality of bimodal distributions. This is shown by

EXAMPLE 2. Suppose that $c(r) = \frac{1}{2}r^2$, and $v_{12} = v_{23} = v_{31} = 1$, while $v_{13} = v_{32} = v_{21} = 16$. It is easy to compute that if population is concentrated in any *two* of these groups (say in the ratio n: $(1-n)$, equilibrium conflict is given by $2\sqrt{n(1-n)}$, which cannot exceed 1. On the other hand, equilibrium conflict in the case of the uniform population distribution over all *three* groups is given by $\sqrt{17/9}$, which exceeds 1. It follows that no bimodal distribution can exhibit maximal conflict.

It is easy enough to see how the example works. Preferences have been chosen so that between any two groups, the effective degree of disagreement is not high. It is true that one party dislikes the issue of the other, but the dislike is not strongly reciprocated, so that equilibrium conflict ends up being low. However, putting all three groups together has the effect of pushing each party to expend more resources, for there is always some other group that dislikes intensely the favored issue of this party.

4.3. *Conflict and Distribution in Contests*

Recall that for a group engaged in a pure contest, utility is derived *only* from its most preferred issue, and no distinction is made, as far as this group is concerned, between the remaining issues. Thus in the case of contests there is no notion of "distance" across the different groups. Issues other than one's own are all equally alien. So in what follows we set $v_{ii} = 0$, and $v_{ij} = 1$ for all i and j such that $j \neq i$.

4.3.1. *Shifting Population Weights and Nonmonotonicity*

We begin by studying the effects of shifts in population weights across groups.

It will be useful to restate the conditions describing equilibrium conflict in a form most suitable for use here. The condition (3) that describes the solution may be restated for contests as

$$\sum_{j \neq i} s_j = \frac{c'(r_i)R}{n_i} \tag{8}$$

for all i, which on rewriting yields the requirement:

$$z_i(1 - s_i) = c'\left(\frac{s_i}{z_i}\right) \tag{9}$$

for all i, where $z_i \equiv n_i/R$.

For the moment, regard z_i as an exogenous variable in (9). In that case, (9) uniquely defines s_i as some function h of z_i (by Assumption 1). Note, moreover, that h must be increasing and twice continuously differentiable (by Assumption 1). The results that we shall describe depend crucially on the shape of h.

Now recall that $(n_1, ..., n_G)$ is really our set of exogenous variables, but that $(z_1, ..., z_G)$ only differs from this set through a "scaling factor" R. With the functional relationship h in place, R adjusts so that the shares s_i sum to unity, yielding an equilibrium (which is unique, as we already know):

$$\sum_{i=1}^{G} h\left(\frac{n_i}{R}\right) = 1. \tag{10}$$

In principle, (10) contains all the information needed to solve for the equilibrium level of conflict, and to relate it to the distribution of the population among the different groups. Indeed, working backwards one might imagine h as implicitly defining a "conflict ordering" on the population vector, whose contours we are supposed to understand. In practice, however, finding a precise description of such an ordering turns out to be a complicated task.

This complexity is perhaps to be expected. We have already commented on the global and nonmonotonic nature of conflict. These persist for contests as well, as we will see.

Consider the simplest of all possible worlds: the case of two groups. Here, the analysis is just a special case of that immediately preceding Proposition 4.2: conflict is always increased by shifting population from the smaller to the larger group, and is therefore maximized when both groups are of equal size.

This simple observation is related to the findings of Tullock [27], provided that population weights are interpreted as the degree of bias favoring one group over another. Thus interpret this model as one where there are only two lobbyists, with the probability that lobbyist i wins the contest being biased by a factor n_i. Tullock argues that increased bias must lower conflict. In the two-group case, which he considers, an increase in bias is similar (though not identical) to making more unequal the population weights.[13] By the discussion above, this lowers conflict.

That the symmetric bimodal distribution maximizes conflict over the set of distributions with $G = 2$ follows from the result that shifting population from a larger group to a smaller group increases conflict. In a more general setting, with $G \geqslant 3$, the repeated application of these population shifts would ultimately produce the uniform distribution with $n_i = 1/G$ for all i.

[13] Tullock does not consider a situation with more than two groups, claiming (correctly) that the situation is mathematically much more complicated. However, it turns out that in general, his intuition works *only* for the two-group case. One source of difference is that with population weights mimicking bias, the population weights enter into the calculation of total conflict as well as in the computation of the winning probabilities.

Can we thus expect the uniform distribution to be a maximizer of conflict for $G \geqslant 3$? It turns out, however, that matters are more complex than this simple conjecture.

We begin our study of the many-group model by considering a special case. Suppose that the cost of effort supply is quadratic: $c(r) = \frac{1}{2}r^2$. In that case a little effort will show that

$$h(z) = \frac{z^2}{1+z^2}.$$

Suppose, furthermore, that there are only three groups. In that case, (10) may be rewritten as

$$\sum_{i=1}^{3} \frac{n_i^2}{n_i^2 + R^2} = 1. \tag{11}$$

With an equal division of population over the three groups, and using (9) with the obvious equilibrium condition that $s_i = 1/3$ for all i,[14] we see that the level of conflict is given by $\sqrt{2/3}$. Denote this level by R_*. Now recall the LHS of (11) with R set equal to R_*, and consider the problem

$$\max \sum_{i=1}^{3} \frac{n_i^2}{n_i^2 + R_*^2},$$

subject to the constraint

$$n_1 + n_2 + n_3 = 1.$$

Suppose that we can show that a symmetric distribution of population is a local strict maximum to this problem. In that case it follows that the symmetric distribution must also locally (strictly) maximize conflict. The reason is that for distributions in a neighborhood around the uniform distribution, it must be the case that (11) must hold with strict inequality ($<$) when $R \neq R_*$. To restore equality in such cases, R must be lowered *downwards* from R_*. This establishes our claim.

Now, it is the case that a symmetric distribution *does* solve this maximization problem (simply set up the constrained problem and check first and local second-order conditions). So we have arrived at the following observation: *the uniform distribution of population locally maximizes conflict in the three-group case.*

[14] Because the equilibrium is unique, it must be symmetric in symmetric situations.

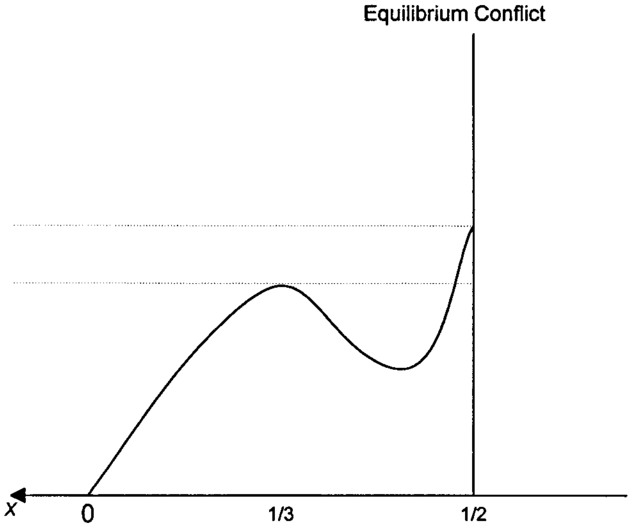

FIG. 2. Conflict in the Three-Group Case.

So far, matters are analogous to our arguments for the two-group case. But the similarity ends there, for the uniform distribution *cannot* be a *global* maximum in the class of three-point distributions! One way to check this is to see that with $\alpha = 2$, the maximal conflict in the two-group case equals 1/2, but that $R_* < 1/2$. By continuity (this can be made precise), there is a three-group distribution close to the uniform two group distribution which exhibits greater conflict than that under the uniform three-group distribution, even though the latter is a local maximizer of conflict!

Figure 2 illustrates this by reproducing the exact behavior of conflict as we move over distributions of the form $(x, x, 1-2x)$. At $x = 1/3$ we encounter a local maximum of conflict. Thereafter, as x declines, conflict falls, only to rise again to an even higher level as x approaches zero. This shows that even in the simplest examples, the relationship between conflict and population distribution is far from obvious.

Is this observation more general? It is. The next proposition summarizes what we know about the uniform distribution for the three-group case, under a wide class of cost functions. To state this proposition, let η denote the elasticity of the marginal cost function: i.e., $\eta(r) \equiv rc''(r)/c'(r)$ for all $r > 0$. Note that by Assumption 1, $\eta(r) > 0$ for all $r > 0$. We make the following technical regularity assumption on this elasticity:

ASSUMPTION 2. The elasticity $\eta(r)$ is bounded, and there is $\delta > 0$ such that for all $r > 0$,

$$-\eta(r)[1+\eta(r)]+\delta < r\eta'(r) < \eta(r)[1+\eta(r)]-\delta. \tag{12}$$

This technical condition is satisfied under a large class of cost functions. For instance, if the cost function exhibits constant elasticity so that $c(r) = (1/\alpha) r^{\alpha}$ for $\alpha > 1$, then $\eta(r) = \alpha - 1$ for all r, so that Assumption 2 is trivially satisfied.

PROPOSITION 4.3. *Suppose that Assumptions* 1 *and* 2 *hold. Then in the three-group case, the uniform distribution of population is a strict local maximizer of conflict. However, there are always other three-point distributions that dominate the uniform in terms of conflict, so that this uniform distribution can never be a global maximum.*

Proposition 4.3 implies that the the creation of bias (via population weights) does not, in general, serve to reduce conflict. It does, in a neighborhood around the three-point uniform distribution, but certainly not everywhere.

A more general result that subsumes Proposition 4.3, can be obtained when there is an arbitrary number of groups. In this case, as the proposition below reveals, the possibility of complicated behavior (in the sense outlined in the previous proposition) depends on the number of groups.

PROPOSITION 4.4. *Take as given any cost function satisfying Assumptions* 1 *and* 2. *Then*

[1] *There exists $\hat{G} \geqslant 3$, such that for all $G \leqslant \hat{G}$, the uniform distribution of population on G groups is a strict local maximizer of conflict, but (unless $G = 2$) it is never a global maximizer. There also exists $\bar{G}$, such that for all $G \geqslant \bar{G}$, the uniform distribution of population on G groups is a strict local minimizer of conflict.*

[2] *For the special case of isoelastic cost functions $c(r) = Ar^{\alpha}$ ($\alpha > 1$), this characterization can be tightened. There exists a decreasing function $g(\alpha)$ with $g(\alpha) > 3$ for all $\alpha > 1$ and $g(\alpha) \uparrow \infty$ as $\alpha \downarrow 1$, such that the uniform population distribution is a strict local maximizer of conflict if and only if $G < g(\alpha)$, and a strict local minimizer if and only if the opposite strict inequality holds.*[15]

[3] *Provided that $G \geqslant 3$, equilibrium conflict is strictly higher at the uniform distribution over $G-1$ groups, rather than at the uniform distribution over G groups.*

[15] We are grateful to an associate editor for improving the statement of this part of the proposition.

Proposition 4.4 continues to counter the idea that an increase in "bias" (induced by unequal population weights) may be conflict-reducing. With larger groups, it becomes more and more likely that the introduction of bias will serve to increase conflict, not reduce it.[16] But the point is that the relationship is a complicated one. This will become even clearer when we argue that *whether or not* a G-point uniform distribution is a local maximizer of conflict, it can *never* be a global maximizer, even in the class of distributions with the same support (see below).

This result also bears on the nonmonotonic nature of the relationship between conflict and distribution, and may be used to supplement Section 6 on the relationship between conflict and polarization. For instance, combine parts [2] and [3] of the proposition. Consider G groups ($G \geqslant 3$), and any cost function such that conflict is locally maximized (the existence of such a cost function is assured). It is then obvious that any "local" departure from the uniform distribution will entail a fall in equilibrium conflict. On the other hand, there are local changes, which if continued in the same vein, will finally lead to the uniform distribution over $G-1$ groups, and therefore an increase in equilibrium conflict. Hence, some distributional changes cannot be broken down in a series of "steps in the same direction," with conflict changing in the same way throughout.

4.3.2. *Group Mergers*

Another implication of the preceding propositions is that a shift of population from larger to smaller groups does not necessarily increase conflict for $G \geqslant 3$. Is there some other partial ordering over the space of distributions which is accompanied by unidirectional changes in conflict? We search, then, for a sequence of conflict-increasing moves that might lead from any starting point to the symmetric distribution on just two groups.

One route to this is by considering group *mergers*. Recall Proposition 4.2, which implies (for contests) that equilibrium conflict is maximized at any symmetric distribution over two groups. Recall also Proposition 4.4, part [3], which states that the uniform distribution over $G-1$ groups is always more conflictual than the uniform distribution over G groups, for $G \geqslant 3$. Both these observations suggest that starting from any initial situation, a *merger* of two groups should raise conflict.

Unfortunately, such an observation cannot be generally true. We show in an earlier version of this paper [6, Propositions 14 and 15] that in the case of a line, group mergers have ambiguous effects on conflict. The same ambiguity persists in the case of contests. But some general results are possible.

[16] Of course, there may be certain ways in which one can introduce bias that will favor a reduction of conflict. But in general, an increase in bias that favors a smaller number of groups will serve to increase conflict and not reduce it.

Proposition 4.5. *Suppose that* $G \geqslant 4$. *Then a merger of any two smallest groups must raise equilibrium conflict. This is also true for* $G = 3$, *provided that the unmerged group is strictly larger than one of the other groups.*

Here, then, is a class of cases in which group mergers unambiguously raise conflict. Of course, this result is requires an upper bound on the size of the merging groups: if they are large, conflict will be reduced.

What about mergers involving a larger number of groups? If the merger involves all but the largest group, conflict increases as well, as the following proposition notes.

Proposition 4.6. *Let* $G \geqslant 3$. *Consider a distribution of the population across G groups. Suppose that* $n_i \neq n_j$ *for some i and j. Then equilibrium conflict is increased by a merger of any* $G-1$ *smallest groups into one. However, if the initial distribution of population is uniform, equilibrium conflict is unchanged.*

These propositions have implications for the phenomenon of "divide and conquer," as can be easily seen by running them backwards. Specifically, starting from the bimodal distribution, if one group is broken up into two or more fragments, all smaller than the intact group, then conflict must go down. It can also be checked that the expected payoff of the intact group will rise.

These propositions also reinforce the nonmonotonicity of conflict. Propositions 4.3 and 4.4 focussed on the absence of a monotonic relationship when population is redistributed across the same number of groups. Here, in contrast, entire groups are merged into one. Imagine that we decompose the merging of $G-1$ *uniformly sized* groups into a sequence of steps in which we merge one group at a time with the already-formed merger. By Proposition 4.5, the merging of the first two groups will strictly increase conflict, provided that $G \geqslant 4$. But the aggregate level of conflict must return again to the initial level after the sequence of steps has been completed, by the second part of Proposition 4.6.

5. CONFLICT AND DISTRIBUTION: PATTERNS

5.1. *Introduction*

When groups engage in lobbying, it is of interest to ask how the "intensity" of lobbying by a particular group might vary with group size. We have already noted that there are reasons to believe that this effect can go either way. The purpose of this section is to provide a systematic analysis.

A natural question is what we mean by the "intensity" of lobbying. We take as an index the share of resources contributed by a particular group relative to its numerical strength in the population. This ratio, s/n, is obviously simply equal to r/R, the *per capita* contribution made by a group relative to the mean.

Patterns of lobbying intensity are related closely to questions of extremism or moderation within a society. To be sure, such concepts are entirely without meaning unless some metric is assigned across groups, so that extremism would then refer to a situation where "radical" groups (in the sense of being outliers in the metric induced by preferences) lobby more intensively for their preferred outcomes. Alternatively, a situation is moderate if "centrist groups" are the most vocal (relative to group size). These notions will be introduced more formally below, but it is worth noting at the outset that such categories are useful in understanding situations where the true distribution of societal characteristics may be exaggerated (or hidden) by its publicly observed conflicts.

As noted above, in the absence of a metric across groups, concepts such as extremism are meaningless. It is nevertheless possible to start with a weaker concept: say that an equilibrium involves *activism* if there are at least two groups i and j with distinct lobbying intensities: $r_i \neq r_j$. It is obvious that the presence of activism, as defined here, is a necessary condition for sharper phenomena such as extremism.

The following proposition completely describes those situations in which activism must be present.

PROPOSITION 5.1. *Equilibria with no activism exist if and only if*

$$n_i \sum_{k=1}^{G} n_k v_{ik} = n_j \sum_{k=1}^{G} n_k v_{jk} \tag{13}$$

for every pair of groups i and j.

5.2. *Activism in Contests*

Proposition 5.1 yields the following straightforward characterization for contests:

PROPOSITION 5.2. *Contests with two groups can never involve activism. On the other hand, contests with more than two groups display activism whenever all groups are not equal-sized, and larger groups always lobby more intensively than smaller groups.*

It may be worth putting Proposition 5.2 in some perspective. Since Olson [20], there has been much debate regarding the effects of group size on collective action.[17] The main point made by Olson has to do with the possibility of free-riding *within* the group, something that is ignored here. So our finding that larger groups lobby more intensively (in contests) does not logically contradict Olson's assertion. However, it does demonstrate that when the issue involved is a public good (rather than a private good whose division is dissipated by size), there are other features of group size that may need to be considered. Indeed, *a priori* it is not at all obvious that larger groups will be activists in our model. The reason is that larger groups are more likely to be confronted by smaller opponents, thus permitting a relaxation of individual effort. The proposition shows that this effect is more than counterbalanced in equilibrium. Nevertheless, a hint of this remains in the statement of the proposition for two groups, where the two effects exactly cancel each other, leading to an absence of activism.

There is another broad class of cases where the straightforward monotonicity result of Proposition 5.2 breaks down. This is when there is a metric over groups, induced by varying preferences for different outcomes. Matters here are somewhat more complicated.

5.3. *Activism on the Line*

We will consider the simplest metric model, that described by a line. For the sake of exposition we study symmetric population distributions on just three groups. Groups are now to be thought of as points on a line segment, identified with issues that are most preferred by the people "located" there. Unlike the case of contests, we shall be assigning externalities to each of the groups should some other group win their most preferred issue. Imagine, then, that there are two groups (whom we shall call the *radicals*), each of size n, situated on either side of a *middle class* of size $1-2n$. Thus $n \in [0, 1/2]$.

It is necessary to describe the losses to one group should another group win. These are given by a (per capita) to the centrist group should either of the radicals win, and by a to either of the radicals should the middle class win. However, should a radical group win, the loss to the other *radical* group is given by b, and we take $b > a$. See below, Figure 3, for an illustration.

Observe that if $b = a$, we are in the case of pure contests. On the other hand, if $b = 2a$, we are in the case of "linear alienation" as studied in [5].

We know by Proposition 3.3 that there exists a unique equilibrium. It follows from this observation and the assumed symmetry of the problem that the two radicals must contribute equal shares (which we denote by s), so that the centrist group contributes $1-2s$.

The presence of a metric across groups (which is deliberately suggested by the terms "radicals" and "centrists") permits us to go further than a

[17] For a summary, see [16].

definition of activism for this case. We may say that a situation is *extremist* if the radicals are activists, and so contribute more than proportionately to their numerical strength, i.e, if $s > n$. A situation is *moderate* if the opposite inequality holds.

It is important to examine when situations might induce extremism or moderation. This gives us an idea of when the true distribution of societal characteristics may be exaggerated (or hidden) by its publicly observed conflicts.

The following result characterizes these situations.

PROPOSITION 5.3. *Define*

$$n^* = \frac{a}{b+2a}. \tag{14}$$

Then a situation is extremist if $n > n^*$, but is moderate if $n < n^*$. It involves no activism if and only if $n = n^*$.

The critical value n^* lies in the interval $(0, 1/3)$, and depends negatively on the excess of b over a. It converges to $1/3$ (the contest case) as $b \downarrow a$, and converges to 0 as $b/a \to \infty$.

In Fig. 3, the distribution of population is shown by the heavy vertical lines, and the magnitude of the equilibrium shares by the lighter vertical lines. The point worth noting is that an already equal society (with a relatively small share of radical groups) will display an even greater degree of moderation in its decision-making, compared to its population distribution. On the other hand, once the population share of the radicals crosses a critical magnitude, then the radicals contribute more than their population share, leading to a situation that looks more conflictual than the underlying population distribution warrants.

Note that extremism manifests itself when each of the radical groups has strictly *lower* population than the middle class: the underlying distribution of characteristics is unimodal. We may record the critical value of n^* for another special case as well: linear alienation. When alienation becomes "convex"—the case $b > 2a$—extremism manifests itself even if the *total* population of radicals is less than the middle class.

Linster [15] noted this result in the special case where $n = 1 - 2n = 1/3$, arguing that extreme groups put in a proportionately larger share of lobbying resources.[18] Radicals have more to lose, so they engage in more conflict.

[18] His model is different in some other minor respects, such as the assumption of a linear cost function.

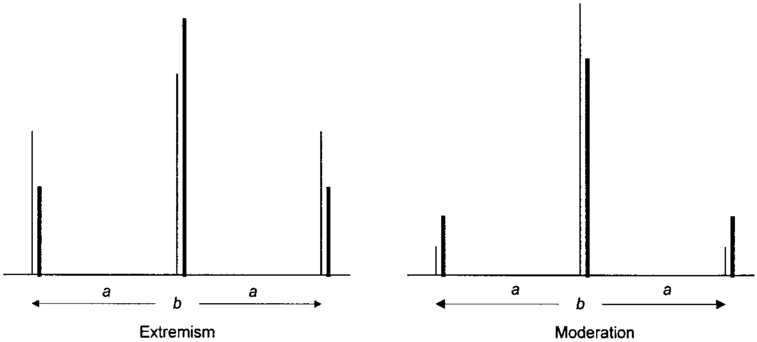

FIG. 3. Extremism, Moderation, and Distribution.

Our results show that the possibilities are richer: relative group strengths also matter. In particular, extremism breaks down when radical groups become very small (a "discouragement effect" takes over).

This special case shows that non-contest situations are potentially more fertile than contests in their implications for activism. We may return once again to the Olson argument, which states that small groups may have more efficacy than large groups. An additional dimension is revealed in the particular scenario studied here. Very small groups are not activist, though moderately small groups are. In contrast to the case of contests, this is more in line with the Olson thesis, though for reasons that are entirely different.

6. LINKS WITH POLARIZATION

We end the paper with some brief remarks on the relationship between conflict and the concept of polarization developed in [5]. Readers who wish to see the analysis in more detail are invited to study [6].

In [5], we introduced the notion of a "polarized distribution" and argued that polarization, not inequality as it is commonly measured, holds the key to our understanding of social tension and conflict. Briefly, polarization is a feature of distributions that combines elements of equality *and* inequality in a particular way. Specifically, we argued that *intra*-group homogeneity, coupled with *inter*-group heterogeneity, lies at the heart of a polarized society, and this feature is correlated with social conflict. At the same time, measured inequality in such a society may be low. The reader is referred to our paper, where the distinction is made clear in a series of examples.

It turns out that conflict, as developed in this paper, moves with distributional characteristics in the broad way suggested by distributional polarization. Three features are worth mentioning.

Property 1: *Bimodality.* There is a two-point symmetric distribution of population which globally maximizes conflict. This result is proved in Proposition 4.2 of the paper. It turns out that bimodality also lies at the heart of increased polarization (Theorem 2 in [5]).

Property 2: *Globality.* Consider the merging of any two groups in a G-group distribution, where $G \geqslant 3$. Then whether conflict goes up or down depends on the sizes of the merging groups, as well as the distribution of the population across non-merging groups. This result is formally established for conflict in [6, Propositions 15, 16]. The corresponding discussion for polarization is Example 4 and Section 3.4.2 in [5].

Property 3: *Nonmonotonicity.* Start with a uniform distribution of population across G groups, where $G \geqslant 4$. Transfer population mass from one of the groups to the others, until a uniform distribution over $G-1$ groups is obtained. Then conflict is higher at the "end" of this process, but may go down in the "intermediate" stages. The discussion in Section 4.3 culminating in Proposition 4.4 bears directly on this issue. For comparison with polarization, see Example 5 and Section 3.4.3 in [5].

There are differences as well. The present model of conflict is more general than the model used to characterize polarization, on two counts. First, we allow for more general metrics over preferences (the analysis in [5] is exclusively relevant for the line). Second and more fundamental, the description of polarization is first and foremost a problem of *measurement* (though it may be motivated, as it was in our case, by a desire to understand conflict), while an analysis of conflict must of necessity rest on a behavioural model.

As an implication of the latter, consider the per-capita effort r_i put in by a typical member of group i. Is this the same across all groups? The answer is no. But a measurement theory, largely devoid of behavioral postulates, will have difficulty predicting this variation. To see this more formally, recall our measure of polarization, adapted here to use the notation $\{v_{ij}\}$ for alienation:

$$P = \sum_i \sum_j n_i^{1+\gamma} n_j v_{ij}, \tag{15}$$

where γ is strictly positive (additional assumptions place further restrictions on γ such as $\gamma \geqslant 1$). Recall that it is the strict positivity of γ that distinguishes polarization from the traditional measures of inequality.

Now let us write down a formula for conflict to compare with (15). This will not be closed-form but nevertheless useful. To do so, assume that we are in the constant-elasticity case: $c(r)=\alpha^{-1}r^{\alpha}$ for some $\alpha>1$. Then the first-order conditions may be rewritten as

$$s_i \sum_j s_j v_{ij} = r_i^{\alpha}$$

for each group *i*. Multiplying both sides by $n_i^{\alpha} s_i^{1-\alpha}$ and rearranging terms, we obtain

$$\left(\frac{n_i}{s_i}\right)^{\alpha} s_i^2 \sum_j s_j v_{ij} = s_i R^{\alpha}.$$

Adding over all groups *i*, and bearing in mind that $\sum_i s_i = 1$, we finally have

$$R^{\alpha} = \sum_i \sum_j \left(\frac{n_i}{s_i}\right)^{\alpha} s_i^2 s_j v_{ij}\,. \tag{16}$$

Now we can compare (15) and (16) in the light of our earlier discussion. Recall that s_i is the equilibrium share of resources devoted to conflict by group *i*, while n_i is, of course, the population share. The divergence between n_i and s_i is thus a measure of the variation in individual lobbying intensity over the different groups. Some of this variation is implicitly captured by the polarization measure, which weights each term by the alienation coefficient v_{ij}. But as recorded in the discussion above, this is not enough.

Thus note that if $s_i = n_i$ for all *i*, R^{α} is identical to the polarization measure *P* for the specific case $\gamma = 1$. Thus we see that it is the *behavioral* nature of conflict, which forces $s_i \neq n_i$ in specific circumstances, which makes it depart from polarization in a significant way.

7. PROOFS

Proof. [Proposition 4] Using (2), the maximization of (1) reduces to choosing r_i to maximize

$$\sum_{j=1}^{G} \frac{r_j n_j}{\sum_{k=1}^{G} n_k r_k}\, u_{ij} - c(r_i),$$

an expression that is well-defined for all r_i because $r_j > 0$ for some $j \neq i$, by assumption. Using the end-point restriction on c in assumption 1, and the fact that utility is bounded, it is clear that the solution to the problem is interior, and that the first-order condition must hold. Simple manipulation then reveals the required necessary condition to be precisely (3). The strict concavity of the function to be maximized will show that (3) must be sufficient as well. By Assumption 1, it will suffice to show that the function $\sum_{j=1}^{G} s_j u_{ij}$ is strictly concave in r_i, with all other r_j held fixed. Verifying this is a matter of simple differentiation.

Proof (Proposition 3.2). Denote by Δ the $G-1$-dimensional unit simplex of resource shares, i.e., the set $\{\mathbf{s} \in \mathbb{R}^G \mid s_i \in [0, 1]$ for all i and $\sum_i s_i = 1\}$. For each $\mathbf{s} \in \Delta$, group i, and $R > 0$, define $q_i(R, \mathbf{s})$ by

$$\sum_{j=1}^{G} s_j v_{ij} = \frac{c'(q_i(R, \mathbf{s}) R}{n_i} \tag{17}$$

if (17) can be satisfied with equality; otherwise, set $q_i(R, \mathbf{s}) = \infty$.

For each $\mathbf{s}$, there is always a group i such that $q_i(R, \mathbf{s}) > 0$ for all $R > 0$. [This assertion is easily verified by recalling our assumption that $v_{ij} > 0$ whenever $i \neq j$.] For such a group, it follows from Assumption 1 that $q_i(R, \mathbf{s})$ is continuous and strictly decreasing in R, with $q_i(R, \mathbf{s}) \to 0$ as $R \to \infty$ and $q_i(R, \mathbf{s}) \to \infty$ as R converges down to the minimum value for which (17) can hold with equality.

On the other hand, if $q_i(R, \mathbf{s}) = 0$ for some $R > 0$, then it is easy to see from (17) that $q_i(R, \mathbf{s}) = 0$ for *all* $R \geqslant 0$. Putting all these observations together, we may conclude that for each $\mathbf{s} \in \Delta$, there exists a *unique* $R(\mathbf{s}) > 0$ such that

$$\sum_{j=1}^{G} q_j(R(\mathbf{s}), \mathbf{s}) n_j = R(\mathbf{s}) \tag{18}$$

Now define the mapping $\phi: \Delta \to \Delta$ by

$$\phi_i(\mathbf{s}) \equiv \frac{q_i(R(\mathbf{s}), \mathbf{s}) n_i}{R(\mathbf{s})}. \tag{19}$$

From the definition of $R(\mathbf{s})$ (see (18)) and the properties of q_i for all i, it follows that ϕ is continuous. By Brouwer's fixed point theorem, there exists $\mathbf{s}^*$ such that $\phi(\mathbf{s}^*) = \mathbf{s}^*$. For each i, define $r_i^* \equiv q_i(R(\mathbf{s}^*), \mathbf{s}^*)$. It is easy to check, using (3), that the vector $(r_1^*, ..., r_G^*)$ constitutes an equilibrium. ∎

Proof. [Proposition 3.3] Suppose, contrary to the statement of the proposition, that there are two equilibria. Let the share vectors under these equilibria be $\mathbf{s}$ and $\hat{\mathbf{s}}$, let R and $\hat{R}$ denote the corresponding levels of conflict, and let $\mathbf{r}$ and $\hat{\mathbf{r}}$ denote the corresponding vectors of resources.

We observe first that $\mathbf{s} \neq \hat{\mathbf{s}}$. For if this were not the case, it follows from (3) that $r_i = r_i'$ for all i, which contradicts the supposition that the two equilibria are distinct.

Without loss of generality, suppose that $\hat{R} \leqslant R$.

Let k be an index such that the ratio $s_k/\hat{s}_k$ is maximized. Observe that this ratio is well-defined, because all equilibria must involve strictly positive share vectors (by Proposition 4). We claim that $r_k > \hat{r}_k$. To see this, note that we must have $s_k > \hat{s}_k$. The claim then follows from the definition of the share vector and the fact that $\hat{R} \leqslant R$.

On the other hand, using the first-order conditions for k, we see that

$$\begin{aligned}
\frac{c'(r_k)}{c'(\hat{r}_k)} &= \frac{\hat{R}}{R} \frac{\sum_{j=1}^{G} s_j v_{kj}}{\sum_{j=1}^{G} \hat{s}_j v_{kj}} \\
&= \frac{\hat{R}}{R} \frac{\sum_{j=1}^{G} (s_j/\hat{s}_j)\, \hat{s}_j v_{kj}}{\sum_{j=1}^{G} \hat{s}_j v_{kj}} \\
&< \frac{\hat{R}}{R} \frac{s_k}{\hat{s}_k} \\
&= \left(\frac{\hat{R}}{R}\right)^2 \frac{r_k}{\hat{r}_k} \\
&\leqslant \frac{r_k}{\hat{r}_k},
\end{aligned}$$

where the strict inequality in the chain above uses the fact that $s \neq \hat{s}$.

Now this inequality, coupled with the observation that $r_k > \hat{r}_k$, means that $c'(r)/r$ cannot be a nondecreasing function. Yet it is easy to see that $c'''(r) \geqslant 0$ implies that $c'(r)/r$ must be nondecreasing, and this is a contradiction. ∎

Proof (Proposition 4.1). We follow closely the argument establishing uniqueness in Proposition 3.3. Suppose, contrary to the proposition, that $R' \leqslant R$.

Let k be an index such that the ratio s_k/s'_k is maximized. Because $R' \leqslant R$, it is obvious that $r_k \geqslant r'_k$.

On the other hand, using the first-order conditions for k, we see that

$$\begin{aligned}
\frac{c'(r_k)}{c'(r'_k)} &= \frac{R'}{R} \frac{\sum_{j=1}^{G} s_j v_{kj}}{\sum_{j=1}^{G} s'_j v'_{kj}} \\
&= \frac{R'}{R} \frac{\sum_{j=1}^{G} (s_j/s'_j)\, s'_j v_{kj}}{\sum_{j=1}^{G} s'_j v'_{kj}} \\
&\leqslant \frac{R'}{R} \frac{s_k}{s'_k} \frac{\sum_{j=1}^{G} s'_j v_{kj}}{\sum_{j=1}^{G} (s_j/s'_j)\, s'_j v'_{kj}} \\
&< \frac{R'}{R} \frac{s_k}{s'_k} \\
&= \left(\frac{\hat{R}}{R}\right)^2 \frac{r_k}{\hat{r}_k} \\
&\leqslant \frac{r_k}{\hat{r}_k},
\end{aligned}$$

where the strict inequality in the chain above uses the fact that $\mathbf{s}' \gg 0$.

This inequality, coupled with the observation that $r_k \geqslant r'_k$, contradicts the assumption that $c'''(r) \geqslant 0$. ∎

Proof (Proposition 4.2). Because $c'''(r) \geqslant 0$, there is a unique equilibrium for any population distribution, by virtue of Proposition 4. Therefore equilibrium conflict is well-defined for any population distribution.

In the discussion leading up to the statement of the proposition, we showed that for any population distribution over two groups i and j, conflict is maximal when $n_i = n_j = 1/2$. Let R^*_{ij} denote the value of conflict under this maximum. If we define $f(r) \equiv rc'(r)$, then R^*_{ij} satisfies the condition

$$f(R^*_{ij}) = \frac{v_{ij}}{4}$$

for all i and j with $i \neq j$ (we use here $v_{ij} = v_{ji}$). For the case of G groups, first note that there exist two groups (say 1 and G) such that $v_{1G} = v_{G1} \geqslant v_{ij}$ for all i and j.

Now consider an arbitrary population distribution over G groups. The first-order conditions (3) tell us that

$$\sum_{j=1}^{G} s_i s_j v_{ij} = f(r_i)$$

for all i. Multiplying both sides of this equation by n_i and adding over i, we obtain

$$\sum_{i=1}^{G} \sum_{j=1}^{G} n_i s_i s_j v_{ij} = \sum_{i=1}^{G} n_i f(r_i). \tag{20}$$

The assumption $c'''(r) \geqslant 0$ implies that f is convex. By a well-known property of convex functions, we may conclude that

$$\sum_{i=1}^{G} n_i f(r_i) \geqslant f\left(\sum_{i=1} n_i r_i\right) = f(R), \tag{21}$$

where R denotes equilibrium conflict for this distribution. Combining (20) and (21), we see that

$$\sum_{i=1}^{G} \sum_{j=1}^{G} n_i s_i s_j v_{ij} \geqslant f(R). \tag{22}$$

Now, observe that for each i,

$$s_i \sum_{j} s_j v_{ij} = s_i \sum_{j \neq i} s_j v_{ij} \leqslant s_i \sum_{j \neq i} s_j v_{1G} = s_i(1 - s_i) v_{1G} \leqslant \frac{v_{1G}}{4}, \tag{23}$$

and combining this information with (22), we may conclude that

$$f(R^*_{1G}) = \frac{v_{1G}}{4} \geqslant \sum_{i=1}^{G} \sum_{j=1}^{G} n_i s_i s_j v_{ij} \geqslant f(R). \tag{24}$$

This proves the existence of a symmetric bimodal distribution which maximizes conflict. To complete the proof, it suffices to note that if any distribution has $s_i \neq 1/2$ for some i, then the very last inequality in (23) must hold strictly for that i. Consequently, (24) must hold strictly as well. ∎

Proposition 4.3 is a special case of Proposition 4.4, which is proved next.

Proof (Proposition 4.4). The following lemma, describing properties of h, will be needed in the proof of this proposition.

LEMMA 7.1. *Suppose that Assumptions* 1 *and* 2 *are satisfied. Then h has the following properties*:

1. *h is strictly increasing and twice continuously differentiable.*
2. *Define $z^* \equiv h^{-1}(\frac{1}{2})$. Then $h(z)/z$ is strictly increasing in z for $z \in (0, z^*)$, and is strictly decreasing thereafter.*
3. *There is $\bar{z} > 0$ such that $h(z)$ is strictly convex on the interval $[0, \bar{z}]$, and*

4. *If* $z^{**} \equiv h^{-1}(\frac{1}{3})$, *then* $h''(z^{**}) < 0$.
5. *If* $(a, b) \gg 0$ *and* $h(a) + h(b) \leqslant \frac{1}{2}$, *then* $h(a+b) > h(a) + h(b)$.

Proof. Recall (see (9) that h is implicitly defined by the function

$$z(1 - h(z)) \equiv c'\left(\frac{h(z)}{z}\right).$$

Differentiating this, and using $s = h(z)$ and $r = s/z$ as shorthand, we see that

$$h'(z) = \frac{r(1+\eta(r))}{s/(1-s) + \eta(r)}. \tag{25}$$

It is clear from Assumption 1 that $h'(z) > 0$ and is continuously differentiable. So part 1 is established.

Next, observe using (25) that

$$\frac{d}{dz}\left[\frac{h(z)}{z}\right] = \frac{h(z)}{z^2}\left(\frac{z}{s}\, h'(z) - 1\right) = \frac{h(z)}{z^2}\left(\frac{1+\eta(r)}{s/(1-s)+\eta(r)} - 1\right).$$

By using part 1 and the definition of z^*, this expression shows that $d/dz\,[h(z)/z] > 0$ when $z < z^*$, with the opposite inequality holding when $z > z^*$. So part 2 is established.

To establish part 3, let $\varepsilon(r)$ denote the elasticity of $\eta(r)$: i.e., $\varepsilon(r) \equiv r\eta'(r)/\eta(r)$ for all $r > 0$. Under Assumption 1, ε is well-defined. Now differentiate (25) with respect to z to obtain, after substantial manipulation,

$$h''(z) = \frac{h'(z)^2}{s(1-s)(1+\eta)[s+(1-s)\eta]} \times \left\{(1-4s+2s^2)\eta - 2s^2 - (1-2s)^2 \frac{\eta}{1+\eta}\,\varepsilon\right\}, \tag{26}$$

where $\eta \equiv \eta(r)$ and $\varepsilon \equiv \varepsilon(r)$.

Denote by $A(z)$ the expression within curly brackets in (26). It will suffice to prove that $A(z) > 0$ for z sufficiently small. Using Assumption 2, and recalling that $s \equiv h(z)$ and $r \equiv s/z$, we see that

$$\begin{aligned} A(z) &= (1-4s+2s^2)\,\eta(r) - 2s^2 - (1-2s)^2 \frac{\eta(r)}{1+\eta(r)}\,\varepsilon(r) \\ &= (1-4s+2s^2)\,\eta(r) - 2s^2 - (1-2s)^2 \frac{r\eta'(r)}{1+\eta(r)} \\ &> (1-4s+2s^2)\,\eta(r) - 2s^2 - (1-2s)^2\,\eta(r) + (1-2s)^2\,\delta \\ &= (1-2s)^2\,\delta - 2s^2[\eta(r)+1] \end{aligned} \tag{29}$$

Now, by Assumption 2, η is bounded, while $\delta > 0$. Moreover, $s = h(z) \to 0$ as $z \to 0$. We may therefore conclude that $A(z) > 0$ for z sufficiently small.

To establish part 4, recall that at z^{**}, $s = h(z^{**}) = 1/3$. Putting $s = 1/3$, manipulating, setting $r^{**} = 1/3z^{**}$ and using Assumption 2,

$$\begin{aligned} A(z^{**}) &= -\frac{\eta(r^{**})}{9(1+\eta(r^{**}))}\left\{\frac{(2+\eta(r^{**}))(1+\eta(r^{**}))}{\eta(r^{**})+\varepsilon(r^{**})}\right\} \\ &= -\frac{\eta(r^{**})}{9\eta(r^{**})(1+\eta(r^{**}))}\left\{(2+\eta(r^{**}))(1+\eta(r^{**}))+r^{**}\eta'(r^{**})\right\} \\ &> -\frac{\eta(r^{**})}{9\eta(r^{**})(1+\eta(r^{**}))}\left\{\eta(r^{**})(1+\eta(r^{**}))+r^{**}\eta'(r^{**})\right\} \\ &> 0. \end{aligned} \tag{30}$$

Finally, to establish part 5, note that if $h(a+b) > \frac{1}{2}$, there is nothing to prove. On the other hand, if $h(a+b) \leqslant \frac{1}{2}$, we see from part 2 that $h(a+b)/a+b > h(a)/a$ and $h(a+b)/a+b > h(b)/b$. It follows that

$$h(a+b) = a\,\frac{h(a+b)}{a+b} + b\,\frac{h(a+b)}{a+b} > a\,\frac{h(a)}{a} + b\,\frac{h(b)}{b} = h(a)+h(b),$$

as desired. ▮

We now return to the main proof. Recall from Proposition 4.2 (and the fact that we are in the special case of contests) that any symmetric bimodal is a global maximizer of conflict, so the first half of part [1] is equivalent to Proposition 4.3. That is, it suffices to show that $\hat{G}$ can be taken to be equal to 3. To prove this, we will use part 4 of Lemma 7.1, which assures us that under the conditions of the proposition, $h''(z^{**}) < 0$, where $z^{**} \equiv h^{-1}(\frac{1}{3})$. It follows that $h(.)$ is locally strictly concave in an open neighborhood around the point z^{**}.

Let $\hat{R}$ be the equilibrium conflict under the three-point uniform distribution. Pick any nonuniform population distribution (n_1, n_2, n_3) such that $n_i/\hat{R}$ lies in the open neighborhood described above, for all i. By local strict concavity of h, and the equilibrium condition,

$$1 = 3h\left(\frac{1}{3\hat{R}}\right) > \sum_{i=1}^{3} h\left(\frac{n_i}{\hat{R}}\right).$$

Let R be the equilibrium conflict under (n_1, n_2, n_3). Then from the fact that h is strictly increasing (Lemma 7.1, part 1) and the above inequality, we see that $R < \hat{R}$.

We turn now to the second half of part [1]. Consider a uniform population distribution over G groups. We know that the equilibrium distribution in this case is symmetric, so that $s_i = 1/G$ for all i. Let $R(G)$ denote equilibrium conflict, and let $z(G) \equiv 1/GR(G)$. Then, using the equilibrium condition (9), we see that

$$z(G)\left(1-\frac{1}{G}\right)=c'\left(\frac{1}{z(G)}\right).$$

Because $c'(0)=0$, it follows immediately from the above expression that $z(G)\to 0$ as $G\to\infty$. So we may choose $\bar{G}$ such that for all $G \geqslant \bar{G}$, $z(G)<\bar{z}$, where $\bar{z}$ is given by part 3 of Lemma 7.1. Let $G \geqslant \bar{G}$. Because h is locally strictly convex in the region $(0, \bar{z})$, we may find an open neighborhood around $z(G)$ such that for every population distribution $\mathbf{n}$ with $n_i/R(G)$ in this neighborhood for all i,

$$1=Gh\left(\frac{1}{GR(G)}\right)\leqslant \sum_{i=1}^{G} h\left(\frac{n_i}{R(G)}\right),$$

with strict inequality holding for every such nonuniform distribution. If R is the equilibrium conflict for any such distribution, then using the expression above and the fact that h is strictly increasing (part 1 of Lemma 7.1), we see that $R > R(G)$.

Next, we prove part [2], which addresses the case of isoelastic cost functions of the form $c(r)=Ar^{\alpha}$. Observe that for such functions, $\eta(r)=\alpha-1$ and $\epsilon(r)=0$ for all r, so that using (28),

$$\operatorname{sgn} h''(z(G))=\operatorname{sgn}[((G-2)^2-2)(\alpha-1)-2],$$

and this is trivially negative for $G=2$ and $G=3$. For $G\geqslant 4$, we see that $h''(z(G))>0$ if and only if

$$\alpha>\frac{(G-2)^2}{(G-2)^2-2}. \tag{27}$$

For each $\alpha>1$, define $g(\alpha)$ as the value of G for which (27) holds with equality (neglect integer restrictions on G). It is easy to see that $g(\alpha)$ is strictly decreasing, that $g(\alpha)\uparrow\infty$ as $\alpha\downarrow 1$, and that $g(\alpha)$ must *strictly* exceed 3 for all $\alpha>1$. Using (27) and the discussion immediately preceding it, we conclude that

$$\begin{aligned} h''(Z(G))<0 &\quad \text{if and only if} \quad G<g(\alpha),\\ h''(Z(G))<0 &\quad \text{if and only if} \quad G<g(\alpha). \end{aligned} \tag{28}$$

But note from earlier arguments that the sign of $h''(z(G))$ is precisely what determines whether the uniform distribution on G groups is a local maximizer or minimizer of conflict; the former in case $h''(z(G))<0$ and the latter in case $h''(z(G))>0$. It follows that $g(\alpha)$ as defined above, and the condition (28), mark the threshold.

To prove part [3], we use (8), and note that $s_i=1/G$ and $r_i=R(G)$ for all i to conclude that

$$\frac{G-1}{G^2}=R(G)\,c'(R(G)).$$

For $G\geqslant 2$, the LHS of the above expression is strictly decreasing in G. Because $rc'(r)$ is an increasing function, it follows that $R(G)$ is monotonically decreasing in G for all $G\geqslant 2$, which completes the proof. ▮

Proof (Proposition 4.5). Let groups 1 and 2 be two smallest groups. As usual, define $z_i=n_i/R$. Then $z_1\leqslant z_2\leqslant z_i$ for all $i\geqslant 3$, so that using the equilibrium condition (10) and $G\geqslant 4$,

$$h(z_1)+h(z_2)\leqslant\frac{2}{G}\leqslant\frac{2}{4}=\frac{1}{2}.$$

So Lemma 7.1, part 2, applies and $h(z_1+z_2)>h(z_1)+h(z_2)$. It follows that

$$h\left(\frac{n_1+n_2}{R}\right)+\sum_{i=3}^{G}h\left(\frac{n_i}{R}\right)>\sum_{i=1}^{G}h\left(\frac{n_i}{R}\right)=1.$$

Consequently, if we denote by R' the equilibrium level of conflict after the merger, and use (10) and Lemma 7.1, part 1, we see that $R'>R$.

The case $G=3$ is established as a special case of Proposition 4.6 below. ▮

Proof (Proposition 4.6). Let group G be a group of maximal size, and consider a merger of groups $\{1, 2, ..., G-1\}$ into 1. Let R and R' respectively be equilibrium conflict before and after the merger. Using Eq. (10), we see that

$$h\left(\frac{1-n_G}{R'}\right)+h\left(\frac{n_G}{R'}\right)=1.$$

Therefore, $R'>(=)R$ if and only if

$$h\left(\frac{1-n_G}{R}\right)+h\left(\frac{n_G}{R}\right)>(=)1. \tag{29}$$

To this end, recall that h is defined by the relationship (9), so that

$$h(z)[1-h(z)] = \frac{h(z)}{z}\, c'\left(\frac{h(z)}{z}\right) \tag{30}$$

Observe that h is strictly increasing (Lemma 7.1, part 1), so that the RHS of (34) is strictly increasing in z. It follows that for every $z>0$, there exists a unique z' such that

$$h(z) = 1-h(z') \qquad \text{and} \qquad \frac{h(z)}{z} = \frac{h(z')}{z'}. \tag{31}$$

Combining the two observations in (31), we see that

$$z' = z\, \frac{1-h(z)}{h(z)}. \tag{32}$$

Applying (31) and (32) to $z = n_G/R$,

$$1-h\left(\frac{n_G}{R}\right) = h\left(\frac{n_G}{R}\, \frac{1-h(n_G/R)}{h(n_G/R)}\right). \tag{33}$$

Combining (29) and (33), it follows that $R' > (=)\, R$ if and only if

$$h\left(\frac{1-n_G}{R'}\right) > (=)\; h\left(\frac{n_G}{R}\, \frac{1-h(n_G/R)}{h(n_G/R)}\right).$$

Using the fact that h is strictly increasing, this implies that $R' > (=)\, R$ if and only if

$$\frac{1-n_G}{n_G} > (=)\; \frac{1-h(n_G/R)}{h(n_G/R)},$$

which is equivalent to the condition

$$s_G = h\left(\frac{n_G}{R}\right) > (=)\, n_G.$$

When all groups have the same population, the above relationship must hold with equality (this follows from Proposition 5.1, proof below). When some group has larger population than another; it must be the case that $s_G > n_G$, so that the above relationship holds with strict inequality (this follows from Proposition 5.2).[19] ∎

[19] Details: Suppose, on the contrary, that $s_G \leqslant n_G$. Then we claim that $s_i \leqslant n_i$ for all i, with strict inequality for some i. This is easy to see by applying Proposition 14, because s_i/n_i simply equals r_i/R for all i. But then $\sum_i s_i < \sum_i n_i$, a contradiction.

Proof (Proposition 5.1). Let ($\mathbf{s}$, R) be an equilibrium. Then the first-order conditions (3) characterize the solution. This equilibrium does not involve activism if and only if

$$s_i \sum_{j=1}^{G} s_j v_{ij} = \delta \tag{34}$$

for some $\delta > 0$ and every i. Lack of activism is also equivalent to $s_i = n_i$ for all i. Using this observation in (34), we obtain (13).

Proof (Proposition 5.2). To verify that contests with two groups can never involve activism, simply note that the condition (13) of Proposition 5.1 is always met in that case. With more than two groups, the requirement (13) reduces to

$$n_i(1 - n_i) = n_j(1 - n_j)$$

for all groups i and j. Because $G \geqslant 3$, it is only possible to satisfy this equality if n_i has the same value for all i.

Finally, we prove that larger groups lobby more intensively. To do so, let us rewrite the first-order condition (8) as

$$g(s_i) = r_i c'(r_i), \tag{35}$$

where $g(s) \equiv s(1 - s)$.

Index groups such that $s_1 \leqslant s_2 \leqslant \cdots \leqslant s_G$. If $s_G \leqslant 1/2$, then, because g is increasing on $[0, 1/2]$, we have that $g(s_i) \leqslant g(s_{i+1})$ for all i, with strict inequality holding if $s_i < s_{i+1}$. On the other hand, if $s_G > 1/2$, then (because $G \geqslant 3$) we see that $s_{G-1} < 1 - s_G < 1/2$, and hence that $g(s_{G-1}) < g(1 - s_G) = g(s_G)$. Therefore, in both cases we have established the fact that $s_i > s_j$ if and only if $g(s_i) > g(s_j)$.

Using (39) and the fact that $rc'(r)$ is strictly increasing in r, we may now conclude that $s_i > s_j$ if and only if $r_i > r_j$.

Finally, note that $s_i > s_j$ if and only if $n_i > n_j$. This is easiest seen by recalling that $s_i = h(z_i)$ (see the paragraph following (9)), and noting that h is strictly increasing and $z_i = n_i/R$. ▮

Proof (Proposition 5.3). To establish (14), use Proposition 5.1 and (13) for one of the radical groups and the centrist group to obtain

$$n^*[(1 - 2n^*)a + n^*b] = (1 - 2n^*)[2an^*],$$

which simplifies right away to (14).

Now suppose that $n > n^*$, but that contrary to the statement of the proposition,

$$\frac{s}{n} \leqslant \frac{1-2s}{1-2n}. \tag{36}$$

Then if r and r' denotes the *per capita* contributions of the radicals and middle class, respectively, $r \leqslant r'$. Using this information along with the first order conditions (3), we see that

$$n[(1-2s)a + sb] \leqslant (1-2n)[2as].$$

Rearranging terms and using (36), we obtain

$$\frac{n}{1-2n} \leqslant \frac{2a}{a(1-2s)/s + b} \leqslant \frac{2a}{a(1-2n)/n + b}. \tag{37}$$

But simplification of (37) yields $n \leqslant n^*$, a contradiction. The case $n < n^*$ is proved in a parallel manner. ∎

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