Collective Action and the Group Size Paradox

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Introduction

Most activities we observe in society — political, social or economic — are carried out by groups or organizations rather than by individuals. Presumably, this is so because in many instances the outcome obtained by pooling efforts is larger than the sum of what is achievable at the individual level. However, the potential advantages of collective action critically depend on the possibility of distributing the benefits from cooperation in ways that pay all potential partners to cooperate.

This is the well known *free-rider problem*. Individual rewards depend on the action contributed by other group members as well as on one's own. In particular, individuals bear only partially the adverse consequences of reducing their contributed effort. Consequently, collective effort typically falls below the group-optimal level.

The free-rider or collective action problem is extremely pervasive and includes a wide variety of situations in which cooperation is necessary. Trade unions, lobbies and the provision of public goods are standard examples. Notice that this problem also appears in the case of collusive behavior between organizations or firms, and even *within* organizations or firms themselves when the results obtained depend in an inextricable way on the efforts by the different individuals at the management or production level.

Olson's celebrated thesis (see Olson (1965) and also the earlier insightful work of Pareto (1927)) argues that the free-rider problem makes *smaller* groups more effective. While individuals always have an incentive to shirk and free-ride on their fellow group members, the effect is more pronounced when group size is large. There are two reasons for this: the larger the group, the smaller is the impact of an individual defection; and moreover — if the prize has any element of privateness

— the larger the group, the smaller the individual prize. Hence, larger groups will be less effective in pursuing their targets. This is in essence the "group-size paradox".[1]

There is general agreement that, given the free-rider problem, individuals will tend to contribute lower levels of "action" (money, effort, time, and so on) the larger the group they belong to. But the key question really has to do with the *aggregate* potency of the group, which is what determines effectiveness in the sense of success probabilities. It should be fairly obvious that decreasing personal contributions are not necessarily incompatible with increasing aggregate effectiveness. Among others, Chamberlin (1974), McGuire (1974), Marwell and Oliver (1993), Oliver and Marwell (1988), Sandler (1992), and Taylor (1987) have pointed out that Olson's proposition of an inverse relationship between group collective action and its size depends on the assumption that the collective good is purely private, so that it can be divided up among the group members. They argue (although do not formally demonstrate) that when the collective good is public in nature — so that individual rewards are fully nonexcludable — Olson's result is reversed: the larger the group, the higher the level of the collective good they will be able to produce. This view — that the Olson relationship depends on whether the good is private or public — constitutes today's common wisdom on the relation between group size and collective action.

These arguments suggest that we should observe interest groups of small size or of large size depending on whether the benefit they are seeking for is private (and divisible) or public (and non-excludable). The theory does not offer sharp predictions for the case of collective goods that produce benefits that are partly public and partly private.

Does evidence conform to this pattern of behavior? Certainly, there is a sense in which Olson's thesis runs counter to the old maxim: *divide et impera*. If smaller groups became more active, the division of one's opponent into a number of smaller units would entail fiercer conflict. Hence, no interest group could welcome the division of their opponent.

As a second example, consider how a firm is organized. As we have already mentioned, in many firms production is joint in the sense that the productivity of each worker depends on the effort made by the others. This gives rise to the same collective action problems within firms and private organizations. This interdependence might not be equally important at all levels. But, it seems to be the general case when we consider employees at the executive level. Large corporations have hundreds of executives. However, we see that an increasing number of firms reward their executives with bonuses tied to the market value of the firm, and that this does nothing to keep firm size small. In fact, casual evidence seems to indicate that (at least until recently with the rise of the dot.com firms) this particular type of reward scheme is found more often in large corporations than in smaller ones (where we could expect that executives could monitor each other more easily). Another example is the case of law firms where successful lawyers are invited to become partners, thus tying their rewards to the effort made by the others. If we are in line with Olson's view, we should not observe this behavior when the benefit to be shared is monetary.

A third interesting source of evidence relates to how individuals organize themselves in order to influence government decisions. We can think (simplistically but usefully, we hope) of the activity of the government as obtaining revenue and then allocating it to a number of activities, following the pressures perceived from the public. A number of government activities have the character of public goods. But many are very close to private goods. Think of transfers either in money or in kind made on the basis of various individual characteristics. These include pensions, subsidies, education and health. Take the most extreme case: transfers in cash. Here, Olson's thesis predicts that we should observe citizens organized in extremely narrow interest groups since it would pay to no one to share with others. An implication is that lobbies would be organized around a very exhaustive (and somewhat artificial) list of "deserving characteristics". For instance, we should see the very poorest organizing their own lobby to become the sole beneficiaries of public support, arguing that those immediately above are not really deserving. In general, there should be extreme splintering of lobby groups, not to mention an artificial narrowing of the issues. We do not seem to observe anything close to this. Whenever an interest group tries to argue in its favor, this is usually (though not always) done by appealing to some general inclusive characteristic. By and large, the tendency is to see individuals organized in quite broad platforms. This sort of evidence does not fully conform to the common wisdom on the relationship between group size and collective action.

In this paper we develop a formal model of collective action that incorporates three features: (i) collective action is undertaken in order to counter similar action by competing groups; (ii) *marginal* individual effort is increasingly costly (as total individual effort rises), and (iii) the collective prize is permitted to have mixed public-private characteristics, where a parameter allows us to run the gamut between pure publicness and pure privateness of the good.

While the first feature is common to a large and growing literature,[2] our exploration of the second feature gives rise to new results of substantial empirical relevance. The main point of the paper is that, if the marginal cost of effort rises "sufficiently quickly" with respect to resources contributed, larger groups will have a higher win probability *even if the prize is purely private.* As we shall argue in the paper, this amounts to an overturning of — and not just a counterexample to — Olson's hypothesis, in circumstances where marginal costs do increase with effort supply.

Notice that the phrase, "larger groups have a higher win probability," is ambiguous.[3] One interpretation is that it applies over the cross section of groups: do larger groups have a higher win probability *evaluated at a single equilibrium*? A second interpretation is more in the nature of comparative statics: at some equilibrium, suppose that the membership of some group exogenously rises. Then is it true that the winning probability of *that* group goes up?

Notice that under the second interpretation, it is necessary — at least in the general equilibrium formulation that we adopt — to track the possible changes in equilibrium magnitudes before we deliver an answer. That is, *other* groups must be permitted to react to this change in group size. Yet we find, remarkably enough, that the answers to the two questions are the same. Larger groups have higher win probabilities under precisely the same (sufficient) restrictions on marginal cost. Thus, without taking sides on which particular interpretation of the Olson argument is to be adopted, the same critique uniformly applies.

We also need to make precise what we mean by the restriction: "marginal cost of effort rises sufficiently quickly", and we do so in the formal analysis. But we wish to emphasize here that the condition can be usefully interpreted, not just as a description of individual tastes and preferences, but as a reflection of institutional structure. For instance, if credit markets are perfect and funds can be borrowed for the purpose of the lobbying process, there is no reason to expect that the marginal cost of such funds will rise with the amount borrowed. To be sure, credit markets are *not* perfect, and even less so if it is known that such funds may be diverted to lobbying. In that case, our marginal cost restriction becomes focal. Viewed in this way, our restriction on marginal costs is a statement about institutions; in this case, about the nature of capital markets.

Following this interpretation for the moment, we see that the less rich the groups in question, the larger should be the marginal cost effect. This is so for two reasons: (a) as already argued, if credit markets are imperfect — and more so for the poor — the marginal cost of money expended will rise more steeply with expenditure, and (b) if effort is expended, not money, then again the marginal cost rises quickly — there are only 24 hours in the day. It follows that in lobby battles that involve relatively low-income groups — such as social services (e.g., education, health, or cash handouts) — we would see large groups forming, in line with our theory. In contrast, when the battles are fought between high-income groups, then the main expenditure is often self-financed money, not personal effort, and indeed the lobby groups that form are narrow and small.

The distinction between money and effort as objects that affect the marginal cost of resources comes out even more clearly in the theory of the firm. With joint production, it would follow from common wisdom that there should only be small firms. But of course, this is not always the case. Large firms not only have bonus schemes for their employees, as we have already mentioned, they even reward them in the opposite of the way implied by Olson's claim. This is in line with our theory, provided that the resources put by employees into the firm are typically non-monetary (so that the rising marginal cost assumption can be fully defended).

Our results are also new in that we provide a unified treatment of the private and public goods cases by allowing for intermediate outcomes in which the outcome is partly public and partly private. This exercise is not for the sake of mere generalization — new insights emerge. These insights have to do with the interplay between publicness of the prize and the increasing marginal cost effect. *Given* some cost function, the existence of some critical degree of publicness is enough to overcome the group-size paradox — this only confirms earlier results in more general form. More significantly, one can prove that the threshold degree of publicness of the outcome is decreasing in the elasticity of marginal cost of effort supply. Indeed, if the elasticity of the utility cost of effort is at least as large as 1, the probability of success increases with group size, *irrespective of the degree of privateness of the prize.*

It should be noted that our — and Olson's — notion of group effectiveness may not be the only variable that influences group formation. We push this qualification a step further in the paper by noting that per-capita *payoffs* to group members are just as important. This motivates a second notion of effectiveness that we briefly explore at the end of the paper.

We end with a qualification. Our setting — as one referee described it — is a "highly abstract model of pluralism", and is not really a model of lobbying in the way most formal political theorists understand it. But reduced-form and black-boxed though it may be, we believe it is the simplest structure within which to analyze group effectiveness, for it captures the idea that government policy is sensitive to group contributions (be they in units of money or effort). Indeed, our paper is set in the same spirit as that of Olson's original argument, with a complete focus on the free-rider problem. We achieve this focus by abstracting from other (possibly more realistic) aspects, but there is no reason to believe that a consideration of those aspects would change the results significantly.

A Model of Collective Action with Free-Riding

The Effort Cost of Collective Action

Suppose that several mutually exclusive alternatives are available to society. Think of these as different locations of a public facility, competing public projects (hospital, library or museum) or different political parties in office. Only one of the available alternatives can come about. Individuals differ in their valuations over these alternatives. All the individuals who rank on top the same alternative form an "interest group". We assume that all individuals with the same favorite alternative are identical.[4]

Let G denote the number of alternatives, as also the number of interest groups. Let N be the total population and $N_1, N_2, ..., N_G$ be the membership of the G groups.

In every collective action problem there are at least two goods involved: the various prizes from collective action and the efforts contributed by individuals to realize their favorite ends. Notice that the structure of preferences over the various prizes may have complex implications. For instance, individuals might free-ride not only on the effort made by their fellow group members, but on the effort contributed by the other groups as well, as long as alternatives have some degree of publicness.[5]

In order to isolate and examine the within-group component of free-riding we remove these aspects of the problem by assumption.[6] In other words, we assume that individuals in any group care most about their favorite alternative, and are indifferent to all other alternatives. The utility differential between the favorite alternative and the remainder (which we denote by w), will drive the arguments below. Note well that this description is compatible with a scenario in which each alternative has a *fully* public component that is symmetrically experienced by all groups.

We shall denote by a the level of effort contributed by each individual. We choose these units of effort deliberately so that effort is *added* across group members to yield group effort. Thus effort may represent dollars, or hours, contributed to the collective cause. This particular interpretation of a is to be maintained in the discussion that follows.

Assume that individual preferences are represented by the (additively separable) utility function

$$u(w, a) = w - v(a), \tag{1}$$

where v is an increasing, smooth, convex function with $v'(0) = 0$, and w, it is to be recalled, is the per-capita benefit from the favorite alternative.

This is equivalent to measuring utility in units of the collective good: from the benefit w we subtract the cost of the effort contributed *translated into the equivalent units of the collective good*.[7] Recalling that we use a as the variable that is directly added over all agents to arrive at win probabilities for a group, the shape of v — in particular, the fact that we allow v to be nonlinear — becomes crucial in what follows.[8] It is therefore useful to discuss the economic meaning of this nonlinearity. To end, consider the individual preferences over (w, a) pairs implied by (1). If v is linear, we are in the case in which effort is directly subtracted from benefits, as in Olson.

There are instances in which this might be a plausible assumption. Think, for instance, of the cost to a lobbying firm of borrowing an extra dollar from a frictionless credit market, so that the rate of interest r is insensitive to the amount borrowed. In this case $v(a)$ is just $(1 + r)a$. However, there are many interesting instances in which this assumption does not appear to be appropriate. This is clearly the case when the collective action is contributed by individuals and consists of personal effort, time, or income. The class of nonlinearities broaden even further if capital markets are imperfect. In these situations it may be more appropriate to assume that additional units of effort are increasingly costly.

Put another way, the *marginal rate of substitution* between reward and effort — the amount of extra benefit that will just compensate an individual for contributing an extra unit of effort — is increasing (as total effort increases). As it turns out, the *rate* at which this increase takes place — that is, the *elasticity* of the marginal rate of substitution with respect to effort — is the key variable that determines the effect of group size in collective action problems.

To quickly formalize this concept, notice that the marginal rate of substitution — call it r — can be simply written as

$$r = v'(a), \tag{2}$$

and its elasticity $\alpha(a)$ at any effort level as

$$\alpha(a) = \frac{\frac{dr}{r}}{\frac{da}{a}} = \frac{av''(a)}{v'(a)}. \tag{3}$$

$\alpha(a)$ will play a central role in our main result.

The Benefits of Collective Action

We turn now to an exploration of the benefits of group action, and to the way in which efforts influence these benefits. We allow for alternatives that have both "public" and "private" components to them, for the simple reason that we wish to examine a view which appears to be common wisdom (at least since Chamberlin (1974)): while individual effort always decreases with the size of the group, *aggregate collective action increases or decreases as the good is purely public or purely private.*[9] To this end, we allow for each alternative to have public or private (monetary) components, which we respectively denote by P (for "public") and M (for "monetary").

For instance, P might represent a socio-economic cause such as "better trade terms for developing countries", "abortion rights", "establishment of a dominant Hindu state", "abolition of affirmative action", "a shorter working day", "saving the dolphins from extinction", and so on. Think of M as a more narrow monetary award, one that it is dissipated with group size: "an additional $1 billion for Medicare", "an additional increase in the U.S. immigration quota", and so on.

We assume that any private, divisible part of the collective good is shared by the N_i members of group i on an equal division basis. Let $\lambda \in [0, 1]$ be the share of publicness in any alternative. Then, the per-capita benefit to each member of group i, provided that alternative i is chosen by society, is

$$w_i = w(\lambda, N_i) = \lambda P + (1 - \lambda)\frac{M}{N_i}, \tag{4}$$

and is normalized to be zero if any other alternative is chosen.

Observe that this formulation posits perfect symmetry of all alternatives, except for the possible dilution of per-capita rewards because of varying group size. Once again, this analytical minimality is a deliberate attempt to focus on the effects of group size alone.

Notice that $\lambda = 0$ corresponds to the case of a purely private, distributable good, while $\lambda = 1$ corresponds to a pure public good with perfect non-excludability. Thus λ serves as a useful parametrization of the degree of publicness in a collective good.

We emphasize that the *perceived* share of publicness in a given collective good will depend on group size. For given λ, P and M, the larger the group, the larger is the perceived share of the public component. We shall denote by θ_i the share of publicness as perceived by an individual member of group i. That is,

$$\theta_i \equiv \theta(\lambda, N_i) = \frac{\lambda P}{\lambda P + (1-\lambda)(M/N_i)}. \tag{5}$$

We now discuss how the choice of alternatives depends on effort. Denote by A_i the total effort contributed by group i. [Recall that this is just the sum of individual efforts within the group.] We assume that the probability of success for group i — which is just the probability π_i that alternative i will be chosen — equals the effort level of group i relative to the aggregate amount of effort A exerted by *all* groups.[10] That is,

$$\pi_i = \frac{A_i}{A}. \tag{6}$$

Therefore, $\pi_i w_i$ is the *expected* value of the collective good to each individual member of interest group i. Notice that under this specification, the marginal return to an additional unit of individual effort (as well as to an additional unit of group effort) is positive but decreasing in effort.[11]

Equilibrium

Expected utility per-capita is given by

$$\frac{A_i}{A} w(\lambda, N_i) - v(a_i).$$

Each individual in each group takes as given the efforts contributed by everyone else in society (including fellow group members), and chooses a_i to maximize expected utility. Our end-point and curvature conditions guarantees that this choice is interior, and that it is completely described by the first-order condition

$$\left[\frac{1}{A} - \frac{A_i}{A^2}\right] w(\lambda, N_i) - v'(a_i) = \frac{1}{A}(1-\pi_i) w(\lambda, N_i) - v'(a_i) = 0. \tag{7}$$

An *equilibrium* is a vector of individual contributions such that (7) is satisfied for every individual in every group. From (7), it is immediate that in any equilibrium, the choice made by each member of any given group will be identical. That is, $A_i = N_i a_i$, so that

$$a_i = \frac{A_i}{N_i} = A\frac{\pi_i}{N_i}. \tag{8}$$

Using (8), we can rewrite (7) as

$$\phi(\pi_i, A, N_i) \equiv \frac{1}{A}(1-\pi_i)\left[\lambda P + (1-\lambda)\frac{M}{N_i}\right] - v'(A\frac{\pi_i}{N_i}) = 0. \tag{9}$$

An equilibrium can now be reinterpreted as a vector of success probabilities — therefore adding up to unity — and a positive number A such that (9) is satisfied for all groups. A is an indicator of scale: it tells us the aggregate amount of collective action that is created in equilibrium.

It is very easy to check that an equilibrium always exists and is unique. Provisionally view π_i as a parameter in (9), and observe that $\phi \to \frac{1}{A}\left[\lambda P + (1-\lambda)\frac{M}{N_i}\right] > 0$ as $\pi_i \downarrow 0$, while $\phi \to -v'(\frac{A}{N_i}) < 0$ as $\pi_i \uparrow 1$. Furthermore, it is immediate that ϕ is strictly decreasing in π_i. Therefore, for each given A and N_i there is a unique value of π_i satisfying (9). In other words, condition (9) implicitly defines π_i as a function of A and N_i: $\pi_i = \pi(A, N_i)$. The equilibrium value of A is then determined by the condition that

$$\sum_{i=1}^{G} \pi(A, N_i) = 1. \tag{10}$$

It is easily seen that $\pi(A, N_i)$ is strictly decreasing in A and varies between 0 and 1. This completes the demonstration of existence and uniqueness of equilibrium.

Group Size and Collective Action

Winning Probabilities

We are now ready to analyze the effects of group size on the provision of collective goods. To this end, we present a pair of propositions. Both results present a sharp contrast to the common wisdom that Olson's result may or may not obtain depending on whether the collective good is private or public. We show that whenever preferences are nonlinear, i.e. $\alpha(a) > 0$, Olson's tenet that the level of collective action diminishes with group size does not generally hold, *irrespective of the degree of privateness of the collective good*. Indeed, if preferences are "sufficiently" nonlinear (in a sense to be made precise below) we show that not only is Olson's result not true, it is exactly reversed.

As already noted in the Introduction, the phrase "larger groups have higher win probabilities" is ambiguous. It might refer to an examination of large versus small groups at some *given* equilibrium. Or it might refer to the implications of increasing the size of some group, in which case the comparison is across equilibria.

Proposition 1 adopts the first of these interpretations.

Proposition 1 *Consider the equilibrium of the game described above. Then,*

[I] *Whenever* $\inf_a \alpha(a) > 1$, *the level of collective action (and therefore winning probabilities) is strictly increasing in group size for all* $\lambda \in [0, 1]$, *i.e. irrespective of the degree of public/privateness of the collective good.*

[II] *More generally, winning probabilities are increasing over a pair of group sizes* n *and* n', *where* $n < n'$, *if*

$$\theta(\lambda, n) \geq 1 - \inf_a \alpha(a) \tag{11}$$

Provided $\inf_a \alpha(a) > 0$ *and the good is not fully private, this condition is automatically satisfied for large enough group sizes. Alternatively, under the same provisions and for any pair of group sizes, it is satsfied for* λ *close enough to unity.*

Proof. We prove the proposition by examining the behavior of π_i over the cross-section of groups, keeping A unchanged at its equilibrium value. The easiest way to do this is to pretend that N_i is a continuous variable in (7), and to differentiate π with respect to N_i. Some tedious calculations reveal that this derivative is given by

$$\frac{d\pi_i}{dN_i} = \frac{\pi_i}{N_i} \frac{\alpha(a_i) - (1 - \theta(\lambda, N_i))}{\alpha(a_i) + \frac{\pi_i}{1-\pi_i}}. \tag{12}$$

Notice that this derivative is guaranteed to be positive when $\inf_a \alpha(a) > 1$, so that part [I] is immediately established.

Condition (11) of Part [II] is also a near-immediate consequence of (12). We only need to observe that $\theta(\lambda, n)$ is a nondecreasing function of n, so that the nonegativity of the derivative evaluated at $N_i = n$ is sufficient. The fact that (11) is satsfied for large group sizes or for λ close to unity (provided that $\inf_a \alpha(a) > 0$) follows simply from the fact that $\theta(\lambda, n) \to 1$ either as $\lambda \to 1$ or as $n \to \infty$. ■

In the next proposition, we investigate what happens when the size of a particular group increases. This involves a comparison across equilibria. Remarkably enough, the results tightly parallel those of Proposition 1.

Proposition **2** *Suppose that the size of one group (of size n) is increased, while all other group sizes are kept unchanged. Then, under the new equilibrium, the winning probability of this group increases if (12) holds:*

$$\theta(\lambda, n) \geq 1 - \inf_a \alpha(a)$$

Just as in Proposition 1, this condition is automatically satisfied for large enough group sizes, or when λ is sufficiently close to unity. In particular, when $\inf_a \alpha(a) > 1$, the winning probability increases irrespective of the degree of public/privateness of the collective good.

Proof. Let group i have size $N_i = n$. Just as in the proof of Proposition 1, use (7) to differentiate π_i with respect to n, keeping A unchanged for the moment. We already know that this derivative is positive if (12) holds. Put another way, we have learnt so far that $\pi(A, n)$ is increasing in n for *fixed* A (assuming (12) applies).

Now we have to allow for the change in equilibrium A. Notice that our exercise so far allows us to conclude that

$$\sum_{i=1}^{G} \pi(A, N_i)$$

has gone up, evaluated at the earlier value of A and N_j, for $j \neq i$, and the new value of N_i. Therefore, using (10), the new equilibrium value of A must increase. But this proves that for every group other than i, equilibrium win probability must

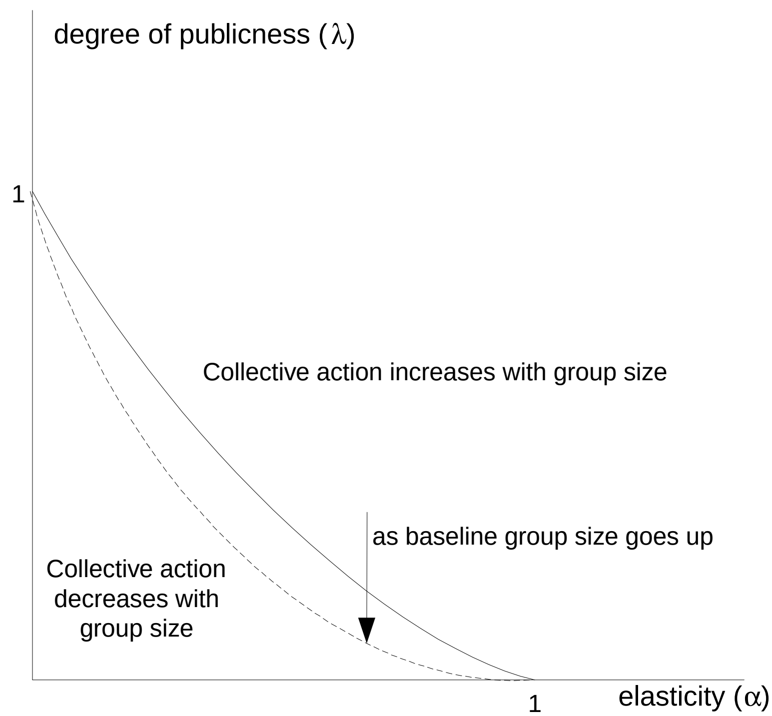

Figure 1: Effectiveness of Large Groups

strictly fall. Because win probabilities sum to one, this completes the proof. [The other particular implications follow just as in Proposition 1.] ■

Assume for the moment that the elasticity $\alpha(a)$ is constant (this will be the case when the effort function is itself constant-elasticity). Then the threshold values of the degree of publicness (λ) for which the Olson result is overturned can be viewed graphically for each value of the elasticity α. Figure 1 displays this. Notice that as α crosses the threshold one, the Olson result is reversed, no matter how private the prize is.

Figure 1 also tells us what happens to this "threshold graph" as the baseline size is increased for the group in question. The graph slides downward, capturing the fact that starting from larger groups, the Olson result is more easily reversed for additions to group size.

Discussion

Propositions 1 and 2 stand in sharp contrast to the common wisdom that collective action increases or decreases in group size depending on whether the collective good is purely public or purely private, respectively. We prove, instead, that the assertion that collective action always varies inversely with group size is false except under either of two extreme assumptions: (i) individual indifference curves are straight lines (i.e. $\alpha(a) = 0$), or (ii) $\alpha(a) \in [0, 1]$ *and* the collective good is *purely* private (that is, $\lambda = 0$).

Indeed, if $\alpha \geq 1$, the Olson assertion is fully overturned. Collective action increases with group size even when the prize is purely private, and this is true *a fortiori* when some degree of publicness is introduced.

Moreover, even if the cost function exhibits lower elasticity (but nevertheless continues to be strictly convex) Proposition 1 argues that there will always be *some* vector of group sizes for which the Olson thesis is false.[12]

In summary, the private/public distinction matters, to be sure, but may be entirely swamped by considerations that involve the disutility cost of effort supply. This sheds new light on the Olson thesis.[13]

This is especially true in the light of our argument that some curvature in the cost function is empirically likely to be the rule rather than the exception.

To obtain further intuition for the propositions, drop subscripts in (12) and rewrite it as

$$\frac{N}{\pi}\frac{d\pi}{dN} = \frac{1}{\alpha(a) + \frac{\pi}{1-\pi}}\left[\alpha(a) - (1 - \theta(\lambda, N))\right].$$

This expression states that the proportional change in the size of collective action induced by a given percent increase in group size is proportional to the difference between the elasticity of the marginal rate of substitution and the elasticity of the value of the collective good with respect to group size.[14] Notice that the reciprocal expression $\frac{1}{\alpha(a)}$ is the elasticity of effort with respect to the marginal compensating reward. Thus, low values of α correspond to extremely elastic responses of effort supply to small variations in the marginal reward. Therefore, the propositions say that collective action will be decreasing or increasing in group size depending on whether the proportional effect on the reward $(1-\theta)$ does or does not induce a sufficiently elastic response in effort supply.

The sign and value of α play a critical role in our result. Which assumptions on α are most plausible? Chamberlin (1974) introduced the distinction between three categories of collective goods: inferior, normal and superior. A collective good is said to be *inferior* if the individual response to an increase by one unit in the effort contributed by the rest of the group is a reduction of the personal effort by no less than one unit, it is *normal* if the reduction is by strictly less than one unit; and *superior* the response consists in *increasing* the effort contributed.

In order to see which would be the assumptions that would give rise to each of these categories of goods, we can differentiate a_i with respect to A_{-i} in the best response function (7), where $A_{-i} \equiv A_i - a_i$. Performing this differentiation and dividing each side by the corresponding side of expression (7), we see that

$$\frac{da_i}{dA_{i-}} = -\frac{1}{1+\alpha(a_i)\frac{A}{2a_i}}.$$

It follows that the collective good is normal if and only if $\alpha(a) > 0$. Therefore, our main result simply requires the collective good to be *normal* in the sense of Chamberlin.[15]

Another Notion of Group Effectiveness

We briefly remark on a second definition of group effectiveness: one that relates group size to per-capita *payoffs*. Observe that this relationship cannot, in general, be predicted by the change in winning probabilities (as described in Propositions 1 and 2). For instance, when the good is purely private, it may be true that larger groups have a higher win probability (if $\alpha > 1$). Moreover, as is well known (Chamberlin (1974)), they also put in lower effort per-capita. However, larger numbers do diminish the per-capita value of the prize. Therefore — holding constant the overall value of the prize — large groups are at some intrinsic disadvantage in terms of payoffs.

To be sure, if the good is purely public, then this disadvantage vanishes altogether and we are simply left with the two positive effects for large groups. This informal discussion suggests that our second notion of effectiveness may be more closely tied to the private/public distinction.

PROPOSITION **3** *In equilibrium, the expected payoff to a player increases with his group size when the collective good is purely public ($\lambda = 1$) and decreases when it is purely private ($\lambda = 0$).*

Proof. Write individual expected utility in an equilibrium as

$$\pi(A, N_i)w(\lambda, N_i) - v(\frac{\pi_i A}{N_i})$$

where A satisfies the equilibrium condition (10). Again, treating N_i as a continuous variable, differentiate this expression with respect to N_i, but keeping A fixed. The idea is that we are moving over a cross-section of groups in a given equilibrium.

Carrying out this exercise and manipulating the results a bit, we see that

$$\frac{\partial u_i}{\partial N_i} = \frac{\pi(A, N_i)w(\lambda, N_i)}{N_i}\left\{\theta(\lambda, N_i) - \left[1 - \frac{1-\pi(A, N_i)}{N_i}\right]\left[1 - \frac{N_i}{\pi(A, N_i)}\frac{\partial \pi(A, N_i)}{\partial N_i}\right]\right\}. \tag{13}$$

The sign of this derivative depends on whether the product of the two square brackets is smaller or larger that $\theta(\lambda, N_i)$. Since $N_i \geq 1$, it is plain that the first square bracket is positive and does not exceed 1. As for the second square bracket, we know —paraphrasing Chamberlin (1974) (see endnote 13) — that it is always positive.

When the collective good is purely private, we have $\lambda = 0$ and $\theta(\lambda, N_i) = \theta(0, N_i) = 0$. Consequently, the derivative in (13) is strictly negative: in equilibrium, members of larger groups attain lower levels of per-capita utility than members of smaller groups.

On the other hand, when the collective good is purely public, we have $\lambda = 1$ and $\theta(\lambda, N_i) = \theta(1, N_i) = 1$. Moreover, by Proposition 1, win probabilities rise with group size. It follows that the value of the second square bracket in (13) does not exceed 1. Consequently, when the good is purely public, the derivative in (13) is positive: members of larger groups attain higher levels of per-capita utility than members of smaller groups. ■

The argument behind this result is quite straightforward. Consider the case of a pure public good. By Proposition 1 we know that, provided it is a normal good, the larger the group, the smaller the individual effort, but the higher the level of collective action. Since the size of the group does not reduce the availability of the collective good to the individual members, belonging to a larger group has the effect of increasing the benefit *and* reducing the costs. Larger groups are more desirable on all counts. When the good is purely private, an increase in the group size still has the effect of reducing individual effort (while the impact on win probabilities is ambiguous). This effect possibly enhances individual utilities, but (as the proposition shows) it is *never* enough to counteract the fall in per capita benefit created by a larger group size.

Combining Propositions 1 (or 2) and 3, we see that there are situations in which collective action undertaken and utilities do not move in the same direction, so that our two notions of group effectiveness are really distinct. For instance, when $\inf_a \alpha(a) > 1$ and the collective good is purely private, larger groups contribute more resources and therefore enjoy larger win probabilities (Proposition 1). However, by Proposition 3, they must have lower payoffs at the individual level.

The reader will notice that Proposition 3 is not as comprehensive as Proposition 1, in that it does not provide a characterization of group utilities in the intermediate cases. Observe, however, that whenever the good is not purely private (that is, $\lambda \in (0, 1]$), θ tends to 1 as N_i becomes large. It can be easily seen from (13) that, for N_i large enough, this expression is strictly positive, so that further increases in group size will increase the equilibrium utility of their members. It follows that whenever the collective good has some public content, large groups may do better (depending on the configuration of group sizes in society).

Concluding Remarks

We motivated this paper by citing several instances in which there is no tendency for small groups to be more effective, *even when the final collective prize is not a purely public good*. The much-examined Olson thesis does not shed full light on this matter. As noted above, the common wisdom concerning this thesis is that it holds in the case of a purely private good, while it may fail in the case where the good is purely public (see, e.g., Chamberlin (1974)).

In contrast, we show that a direct study of the costs of contribution brings out a new aspect of this problem. If *marginal* costs rise sufficiently fast with contributions, then not only is the Olson thesis false (in the sense that a counterexample may be provided), but it is fully overturned. *Even when a good is purely private*, large groups have higher win probabilities than their smaller counterparts (Propositions 1 and 2). This brings out the extreme dependence of the Olson result on the linearity of cost functions, an unrealistic assumption that we question in the paper.

It is true, however, that when a good is *fully* private, larger groups have lower per-capita payoffs than their smaller counterparts, if we control for the overall size of the prize (Proposition 3). While this payoff-based notion of effectiveness is different from the win-based notion that has received attention, it is certainly to be considered in theories of group formation.

While we do not pursue this line of reasoning here, some remarks may be useful. First, it is unclear whether group formation occurs on the exclusive basis of per-capita payoffs, or whether effectiveness in the sense of win probabilities (the perception of being successful) has also something to do with it. If we return to an earlier example and view firms as an instance of groups, we see that this ambiguity is closely related to the age-old question of what firms "maximize": is it simply profits, or size and presence, other public perceptions of success, or some combination of these? To the extent that these other factors also matter, win-based notions of effectiveness also enter into the theory of group formation, and there is no guarantee that society will be splintered into small Olson-style lobbies.

Second, even if per-capita payoffs are the *only* criterion for group formation, Proposition 3 (unfortunately) throws little light on the question of small versus large group selection. The reason is that the proposition is only unequivocal in the "edge cases" of pure privateness or publicness. Just how much privateness is required (under this criterion) for the small-group effect to dominate remains an interesting and open question.

Finally, it is important to remember that in many cases, groups are defined by their ideal points, and there may be little or no room for group *formation.* So there is no necessary contradiction between the possibility that larger groups enjoy low per-capita payoffs and the fact that such groups exist in the first place.

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Notes

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[1]Olson (1965), pp. 36 and 38, observes that "the larger the group, the less it will further its common interest (...) large groups will fail, small groups will succeed." Pareto (1927, p. 379) noted that "(a) protectionist measure provides large benefits to a small number of people, and causes a very great number of consumers a slight loss. This circumstance makes it easier to put a protection measure into practice."

[2]In his analysis of the problem of collective action, Chamberlin (1974) emphasized that most of the theory of collective goods has dealt with actions of a single group and concludes that the next step should be the consideration of the behavior of several groups in competition. The question is of intrinsic interest for, as Hardin (1995) points out, successful collective action, in many instances, comes from the suppression of another's group interest. Therefore, in order to understand group behavior it is essential to model it in the broader frame of the competition among various groups with conflicting interests. A sizeable literature has emerged on this subject. See, e.g., Katz, Nitzan and Rosenberg (1990), Nitzan (1991), Riaz, Shogren and Johnson (1995) and Tullock (1980), who formalize the problem of

collective action with multiple groups as a contest game.

[3]We are indebted to an anonymous referee for bringing this point to our attention.

[4]We thus exclude the case of group members who might be prepared to supply by themselves the necessary contributions. This particular question was used by Olson to classify different types of groups and has been extensively analyzed by Marwell and Oliver (1993). Without denying the potential empirical relevance of this issue, we abstract from it in our desire to focus on group numbers alone.

[5]To illustrate this point, consider the case of three groups and three alternatives, each set labeled A, B and C. Suppose that members of group A, strongly prefer alternative B to C if choice is restricted to this pair. Suppose, moreover, that that members of group B are pushing very strongly for their most preferred outcome. Consequently, when deciding how much effort to contribute, individual members of group A would take into account not only what their fellow-members are contributing, but also the fact that alternative B is not that bad. Therefore, the free-riding problem could not be confined to one's own group, but would spill over to the effort made by "nearby" groups.

[6]The case of more complex inter-alternative preference structures has been explored in detail in Esteban and Ray (1999). There, we examine the complementary case in which there is no within-group free-riding, but groups may free-ride each other. We show that, in general, the structure of preferences over the entire set of alternatives has implications for the magnitude and pattern of conflict. However, in the interest of providing a narrowly focused examination of the Olson argument,

we avoid the general case in this paper.

[7]Note that a could as well be contributions in money. In this case, $v(a)$ is the utility cost of the contributed amount of money.

[8]Taylor (1987) points out that Olson's argument critically depends on the assumption that costs can simply be subtracted from the benefits, and mentions that Olson was aware of this analytical weakness (see Olson (1965), p. 29 note 46). We shall show this is an important observation with implications far stronger than the ones derived by Taylor through his diagrammatic analysis.

[9]See the summary of the "state of the art" in Taylor (1987, Chapter 2) or in Sandler (1992, Chapter 2).

[10]The model can easily be rewritten with success probabilities depending nonlinearly on individual effort (see Skaperdas (1996)), but in this case the interpretation of the nonlinearity of v becomes more complex.

[11]Marwell and Oliver (1993) use the term "decelerating" to describe this phenomenon.

[12]Indeed, if $\alpha(a) \geq \frac{(1-\lambda)M}{\lambda P+(1-\lambda)M}$ the Olson thesis is false regardless of the configuration of group sizes in society.

[13]To be sure, the amount of *individual* effort contributed diminishes with group size for all $\alpha(a) \geq 0$ and all $\lambda \in [0,1]$, a result known since Chamberlin (1974); see also Riaz, Shogren and Johnson (1995).

[14]To see why $1-\theta(\lambda, N)$ serves as a measure of the latter elasticity, consider

the percentage variation in per capita reward $w(\lambda, N)$ corresponding to a given percentage increase in population size. It is easy to see that $\frac{\partial w}{w} / \frac{\partial N}{N} = -\frac{N}{w}(1-\lambda)\frac{M}{N^2}$ $= -\frac{(1-\lambda)(M/N)}{w} = -[1-\theta(\lambda, N)]$. Therefore, the degree of perceived privateness $(1-\theta)$ can also be interpreted as the elasticity of w — the reward — with respect to group size.

[15]This result represents a substantial exploration of the observation made by Taylor (1987, Chapter 2) regarding the critical role played by the flatness of the individual indifference curves between reward and effort.